

## PHYSICALLY-BASED MODELS FOR LIQUID SOUNDS

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### ABSTRACT

A physically based liquid sound synthesis methodology is developed. The fundamental mechanism for the production of liquid sounds is identified as the acoustic emission of bubbles. After reviewing the physics of vibrating bubbles as it is relevant to audio synthesis, a sound model for isolated single bubbles is developed and validated with a small user study. A stochastic model for the real-time interactive synthesis of complex liquid sounds such as produced by streams, pouring water, rivers, rain, and breaking waves is based on the synthesis of single bubble sounds. It is shown how realistic complex high dimensional sound spaces can be synthesized in this manner.

### 1. INTRODUCTION



Figure 1: A photograph of a water drop falling in water. Photo courtesy of Andrew Davidhazy.

The sounds made by liquids, especially those of water, are prevalent in our environment and are, or should be, an

integral part of many simulation/animation environments. A wide variety of sounds such as splashes, pouring, streaming, breaking waves, and dripping are easily identified by humans as liquid sounds, yet they seem to lack a simple common auditory feature.

Because of the great variety of sounds in this class, it is very difficult to create these sounds synthetically, or even manipulate recorded sounds in a physically meaningful manner. No physically-based audio synthesis method for liquid sounds has been developed thus far, and for sound effects of liquids one still has to rely on sound recordings.

If these sounds are to be used in interactive environments such as games, immersive environments, or data sonifications, a real-time parameterized liquid sound synthesis method is necessary. This requires first an understanding of the fundamental physical processes involved in sounds production by liquids, and second an understanding of the manner in which these processes combine in diverse complex realistic scenarios. Some environments like streaming rivers are clearly stochastic in nature, whereas others such as dripping water are more deterministic in their time domain structure.

A “brute force” approach to the problem of finding a physically based synthesis method would be to synthesize the sound made by water by numerically solving the compressible Navier-Stokes equation for water and air at audio rates, i.e., with time steps of about  $1/44100s$ .

It is instructive to compare this with the physically based synthesis of realistic contact sounds. The “brute-force” method was used in [1] by running a simulation of deformable bodies with contacts at audio time steps. The sounds were synthesized from a purely physical model, but very long computation times were required to render even short sound effects in this manner. In a simultaneous publication [2] a real-time synthesis system was proposed using modal resonance models [3] for solid objects. Within this method the graphics and animation simulation proceeds at normal time steps and specialized efficient audio algorithms render the audio in real-time, allowing for interactive high quality contact sounds. In a subsequent publication [4] it was shown how an offline precomputation of the rigid body vibrations

can produce realistic modal models to be used within the real-time system described in [2].

Despite recent advances in liquid simulation [5, 6, 7, 8] in animation and computer graphics, here the “brute-force” method seems to be too demanding at present, even for off-line rendering. This is because liquids have many more degrees of freedom than rigid bodies, and the equations describing the motion are nonlinear.

The approach pursued here is to model audio generation by liquids using specialized physically-based algorithms which run at the audio rate. The audio synthesis is intended to be driven by control algorithms which run at much larger time steps. The audio synthesis process can then run simultaneously with either an animation of a simulation based visual rendering of water, and perhaps even allow interactivity, such as splashing water and hearing the correct sounds interactively. This could possibly allow the realistic synthesis of moderately complex sounds such as made by a water droplet as depicted in Figure 1.

The remainder of this paper is organized as follows. In Section 2 the physical processes responsible for the creation of liquid sounds are explained. In Section 3 the physics of acoustic bubbles is reviewed. In Section 4 a physically based model of bubble sounds is constructed. In Section 5 statistical models of complex water sounds are explored and a real-time liquid sound synthesizer is described. An exploration of the sounds that can be created with the parameters of the liquid simulator is presented in Section 6. Conclusions are presented in Section 7.

## 2. PHYSICS OF LIQUID SOUNDS

What physical principle is responsible for the sounds made by water? It has been known for a long time [9] that water by itself hardly makes any sound at all. It is only when air is trapped by water in the form of bubbles that sounds are heard. For example, when a drop of water falls on a water surface, there is a very soft impact sound, followed by some more or less pitched sounds caused by bubbles in the water.

The impact sounds are primarily generated by supersonic shockwaves right after impact. If one assumes a falling drop to be spherical in shape and the water surface to be a plane, there is a brief period after initial contact during which the contact region moves with supersonic speed, which creates a small shock wave [10].

Far more important appear to be bubble sounds. Bubbles are formed when the water surface causes air to be trapped in the water, or when air is injected in the liquid by a nozzle, by blowing a straw in a glass of water or some such mechanism. Cavitation is another source of bubbles which has been studied extensively as it is an important factor in many engineering problems such as building high speed quiet submarines.

Bubble formation is usually accompanied by an energy injection into the bubble at creation time. This can happen for example when a surface wave breaks and traps bubbles, when a cavity created by a falling droplet collapses, or when a parent bubble breaks up and surface tension excites the child bubbles.

After formation, the bubble emits a sinusoidal sound which decays as energy is dissipated. If the bubble survives long enough, this is all that happens. The bubble may also break up into smaller bubbles, which are excited by surface tension forces at the time the bubble breaks up. This process would result in a pair of bubble sounds immediately after the original bubble sound. If the bubble is formed close to the water-air interface and is rising, the pitch of the bubble rises, giving the familiar “blooink” sound that is audible sometimes when a stone is thrown in water, and the appropriate cavity is formed (which is an intermittent phenomena).

The acoustic mechanism responsible for bubble sounds is volume pulsation, which was first correctly identified by Minnaert [11]. One way to visualize this is to think of the bubble as a small compressible region added to the incompressible fluid, allowing it to oscillate. In this interpretation the water behaves as an effective mass and the bubble as a spring. Another viewpoint is to consider the bubble as a source of underwater sound which propagates through the liquid.

Other mechanisms for the production of bubble sounds proposed previously are resonances in the bubble air mass [12], and shape oscillations of the bubble. These processes can be readily excluded from being responsible for audible liquid sounds, as it is not difficult to show that they lead to frequencies that are either too low, or too high to account for observed sounds.

The physics of a single spherical bubble is well understood, and Leightons monumental book “The Acoustic Bubble” [13] provides an extensive source of information on this.

## 3. BUBBLE PHYSICS

The material in this Section is mainly derived from [13]. The impulse response  $\iota(t)$  of a radially oscillating bubble is given by

$$\iota(t) = a \sin(2\pi ft)e^{-dt}, \quad (1)$$

where  $f$  is the resonance frequency,  $d$  is the damping factor,  $a$  is the amplitude, and  $t$  is time. The function  $\iota(t)$  represents the deviation from equilibrium of the bubble radius and is assumed to be small compared to the equilibrium radius  $r$ .

The resonance frequency  $f$  of a bubble in an infinite volume of water is approximately given by Minnaerts formula,

which under normal atmospheric conditions takes the form

$$f = 3/r, \quad (2)$$

with  $r$  the bubble radius in meters. Since the human ear can not hear frequencies above  $20000Hz$  or below  $20Hz$ , this means that for audio purposes we only have to be concerned with bubbles of radii from  $15cm$  to about  $0.15mm$  at most.

The energy loss of a resonating bubble due to viscous, radiative, and thermal processes can be computed by straightforward thermodynamics, and results in a damping parameter  $d$  given by

$$d = 0.13/r + 0.0072r^{-3/2}, \quad (3)$$

which is valid for bubbles larger than  $0.15mm$ , for which viscosity effects do not contribute significantly to the energy loss during sound emission. The amplitude  $a$  depends on the excitation, which is determined by the bubble creation mechanism. A simplified model of bubble formation due to the impact of a droplet or a solid into the water is drawn schematically in Figure 2. At the moment the tunnel created by the impacting droplet collapses, the air in the bubble is enclosed and the kinetic energy of the water due to the inward motion is used to compresses the bubble. In reality a droplet impact is a very complicated phenomenon and often a jet of water is emitted which breaks up into secondary droplets which create bubbles of their own. A similar mechanism due to the breaking of a small wave or cusp is sketched in Figure 3.

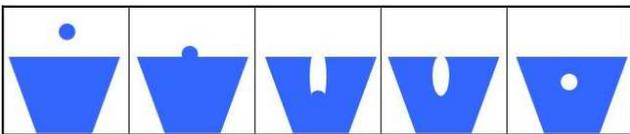


Figure 2: Cartoon drawing of the formation of an underwater bubble after the impact of a drop.



Figure 3: Cartoon drawing of the formation of an underwater bubble by the breaking of a very small wave or cusp, possibly caused by shaking the container or stirring the water with an object.

An analysis of the kinetic energy in the fluid around a spherical bubble shows that the energy  $E$  injected into a bubble is

$$E = 2\rho r^3 u^2,$$

where  $\rho$  is the liquid density,  $r$  is the bubble radius, and  $u$  is the average inward normal velocity at the boundary of the bubble. This leads to a prediction for the amplitude  $a$ , namely

$$a \approx r\sqrt{ru}. \quad (4)$$

The submerged oscillating bubble will thus create a sound, which propagates to the surface of the liquid where it is transmitted to air. The attenuation due to the propagation and transition to air is in principle straightforward to compute if the configuration of the water and the location of the bubble is known.

In many real-world situations a bubbles rises to the surface while emitting sound, resulting in a noticeable increase in frequency over its acoustic lifetime. In the Minneart model the spherical shell of water around a deeply submerged bubble acts as an effective mass. If the bubble rises to the surface, this effective mass is reduced. It can be shown that the frequency of a bubble just under the surface will have a frequency  $\sqrt{2}$  higher than a bubble that is completely submerged. As a rule of thumb, a bubble that is deeper than  $10r$  under the surface can be considered to be fully submerged.

Informal examinations of spectrograms of the sounds of droplets falling into water readily shows the existence of these rising bubbles, which have a characteristic familiar sound. They occur intermittently, when the conditions are just right. The observed rise in frequency is usually around an octave or less for bubbles created by droplets falling into water. Figure 4 shows a series of bubble sounds created by a droplet falling into a body of water. The rising pitch can be seen. By transposing the recording down by a few octaves the characteristic rising bubble sound is very clearly audible. Much larger and faster rises in frequencies occur when air is injected through a nozzle into water close to the surface. When a sound emitting bubble is partly submerged its effective mass reduces much more than when it is just close to the surface, and this process happens rapidly when the bubble reaches the surface. Figure 5 shows a spectrogram of the sounds of bubbles being blown in water in this manner.

#### 4. SINGLE BUBBLE SOUNDS

The perceptual characteristics of liquid sounds must be embodied in the acoustical properties of the bubbles as explained in the previous Section. For individual bubble sounds, there seem to be two factors which could allow us to identify an exponentially decaying sinusoid as a bubble sound.

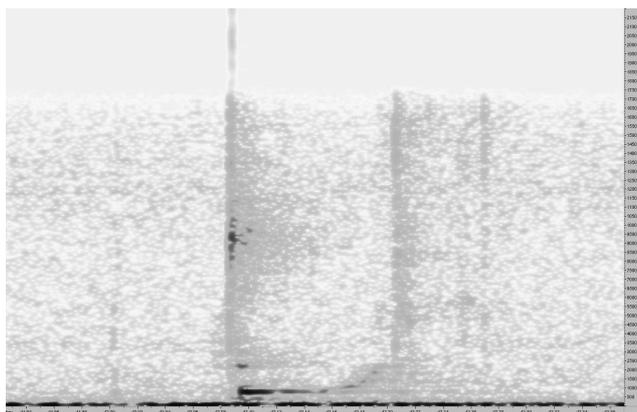


Figure 4: Spectrogram of the sound of a water droplet falling from a height of about 1m into a toilet bowl. There is a rise in pitch towards the end.

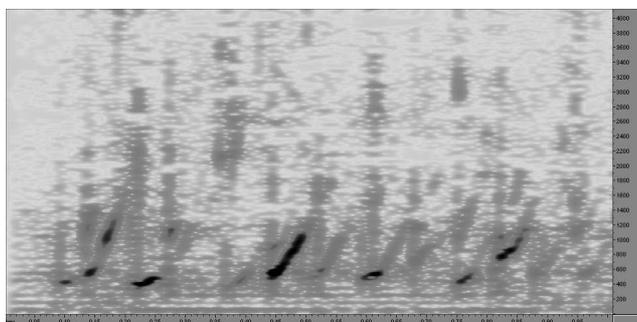


Figure 5: Spectrogram of the sound of blowing with a straw into water close to the surface. The rise in frequency is much larger and faster than it is for bubble sounds created by falling drops.

First there is the relation between the frequency and damping of the bubble sounds. From Eq. 2 and Eq. 3 it follows that the relation between frequency and damping is given by

$$d = 0.043f + 0.0014f^{3/2}. \quad (5)$$

It is interesting to compare this to the corresponding relation between frequency and damping for sounds made by solids, whose audible material properties seem to be encoded predominantly by the relation between frequency and damping [14], which is approximately linear of the form

$$f = Cd.$$

The constant  $C$  is characteristic of the material that is heard as was shown by user studies in [15].

The second perceptual cue that we are hearing a bubble sound could be the rising pitch of a bubble formed close

enough under the surface. The rising bubble is modeled by making the frequency time dependent according to

$$f(t) = f_0(1 + \sigma t),$$

where  $f_0$  is the Minnaert frequency and  $d$  the damping. The factor  $\sigma$  is the slope of the frequency rise, and is related to the vertical velocity of the bubble. A perceptually more relevant parameter is the audible rise in pitch, which also depends on the damping factor  $d$  of the bubble sound. By writing  $\sigma = \xi d$ , we take the effect of damping into account and  $\xi$  roughly parameterizes the audible rise. For sounds associated with drops a value of roughly  $\xi = 0.1$  seems right. Smaller values of  $\sigma$  are aurally acceptable of course, corresponding to slower and/or deeper bubbles. Higher values do not sound anymore as bubbles associated with water drops, but sound more like bubbles created by blowing in a straw with the end very close to the water surface. The rising bubble in Figure 4 has a  $\xi$  factor of about  $\xi = 0.1$  and the middle bubble in Figure 5 has a  $\xi$  factor of roughly  $\xi = 10$ .

To test the realism of bubble sounds modeled by the impulse response of the Minnaert model, a number of single bubble sounds were created with diameters (in mm)

$$r = [10, 7, 4, 2, 1, 0.5, 0.3],$$

each with  $\xi$  factors  $\xi = 0$ ,  $\xi = 0.05$ , and  $\xi = 0.1$ . Subjects were asked to rate the realism of the synthetic sounds compared to “the sound of a water drop” on a scale of 1 – 5. In the real world single bubble sounds can be created by droplets of water falling into a body of water, though usually even this simple event is a complex phenomena and many bubbles and secondary droplets occur. Nevertheless, with some skill it is possible to set the conditions for a droplet such that a single bubble sound is heard.

The bubble sounds are available on [16]. The results of 19 subjects are depicted in Figure 6.

The results suggest that bubbles with radii in the range 2 – 7mm are most readily associated with the sound of a water drop. It also seems that the rising pitch increases the realism of the larger bubbles (4mm and up), but does not have much effect on the smaller drops. This is probably because the smaller bubbles have very high pitch (3000Hz and up) and decay very rapidly so the rise in pitch is difficult to hear.

The sounds of small bubbles are heard as impacts without any particular liquid signature. They could be interpreted as the sound of small drops falling on a hard surface. Very large bubbles sound distinctly unnatural. This does not mean that the Minnaert model fails here, but rather that larger bubbles do not occur in isolation in nature. These larger bubbles are created when for example a large stone is thrown in the water. The large cavity is however accompanied by many secondary splashes and this is probably the

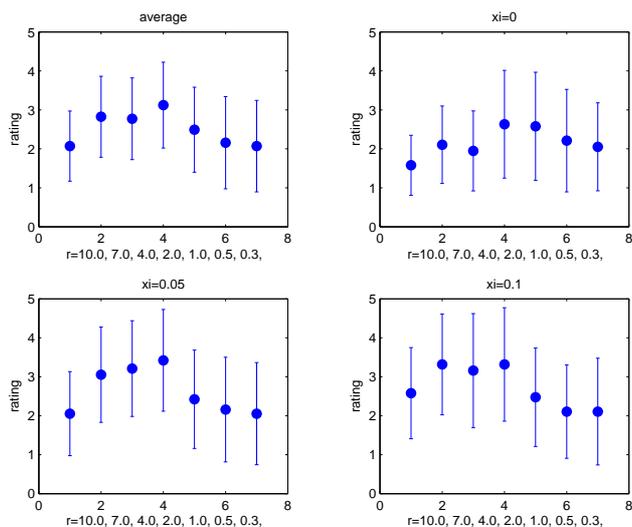


Figure 6: The subjective rating of the realism of synthetic drop sounds on a scale of 1 to 5 by 19 subjects for 3 values of  $\xi$  and the average of those. The average over subjects and the standard deviation are shown.

reason that the sound of the large principal bubble by itself does not elicit a liquid sound perception. The sounds of larger bubbles, with  $\xi$  values around 0.5 – 1.0 sound somewhat like large air bubbles produced underwater close to the water surface by blowing air through a nozzle.

In reality, bubble sounds are almost never heard in isolation. Even a relatively simple event such as a single drop falling in water already generates a complicated sequence of bubbles and secondary droplets, as can be seen in Figure 1. In theory it should be possible to use a liquid simulator which solves the Navier-Stokes equation, use it to compute bubble formation and excitations, and produce realistic audio in this manner. This is a promising area for future research and is suitable for more or less deterministic simple processes such as a droplet falling into water, which produces few bubbles, with distinct sounds.

### 5. COMPLEX LIQUID SOUNDS

More complex phenomena such as streams, pouring water, rivers, rain, and breaking waves generate huge quantities of bubbles and a statistical approach seems warranted. For these sounds the individual characteristics of the bubble sounds themselves are merged into the impression of a continuous sound stream. In [17] statistics on the bubble population of several small rivers and streams was collected. In [18] droplets were photographed with a high speed camera and the sounds recorded simultaneously to observe correlations between bubbles and sounds. The sound of rain

and its statistical properties which relate to bubbles has been studied experimentally [19, 20, 21].

While the statistical properties of these complex phenomena are in principle computable from simulations, leading to a fully physical model for liquid sounds, in this paper we present a hybrid approach and combine the physically based sound model for the individual bubbles with an empirical phenomenological model for the bubble statistics. This synthesis approach could be described as Physically Informed Sonic Modeling [22] by granular synthesis [23].

As a design tool, a bubble simulator has been created which allows the real-time synthesis of a population of a large number of active simultaneously sounding bubbles, modeled using the Minneart model and excited according to stochastic processes. The implementation presents a slider-based interface allowing the user to set the range of bubble sizes to be modeled, the average number of bubbles per second, the fraction of bubbles with a rising pitch, and the distribution of the bubble population over the different bubble sizes. The interface is depicted in Figures 7 and 8. Various preset distributions can be loaded, or the user can set the distribution manually. All manipulations of the synthesis parameters can be done in real-time so it can be used as a tool to explore the sound space of bubbles in an interactive manner.

The bubble simulator is written in pure Java using JASS [24] for the audio synthesis core and the graphical user interface. It is available from [16] and runs on all platforms supporting Java.

The minimum and maximum bubble sizes of the population are adjustable with the bottom two sliders to cover a maximum range of 0.2 – 50mm. A bubble population is created consisting of  $N = 50$  bubbles, logarithmically distributed over the user defined range of radii  $[r_{\min} r_{\max}]$ . Their radii  $r_k, k = 1, \dots, N$ , are given by

$$r_k = r_{\min} \left( \frac{r_{\max}}{r_{\min}} \right)^{\frac{k-1}{N-1}}.$$

The creation of a bubble with radius  $r_k$  is modeled as a Poisson process with expected time intervals  $\tau_k = 1/\lambda_k$ , where  $k = 1, \dots, N$  labels the bubbles. The rate of bubble creation is  $\lambda_k$ . The  $\lambda_k$  values can be controlled using a window with  $N$  sliders, which sets the relative rates  $\bar{\lambda}_k$ . These are related to the actual rates by

$$\lambda_k = \Lambda \bar{\lambda}_k / \sum_{k=1}^N \bar{\lambda}_k,$$

where  $\Lambda$  is the total bubble creation rate, which is also adjustable with a slider. This is done to allow the user to control the total bubble creation rate separately from the distribution of rates over the different bubble sizes. The gain factor  $a_k$  as it appears in Eq. 1, is taken to be

$$a = Dr_k^\alpha \tag{6}$$

where the exponent  $\alpha$  is adjustable with a slider, and  $D$  is a factor related to the depth of the bubble which is discussed below. If we assume the inward radial fluid velocity at the bubble creation time to be independent of the bubble size, Eq. 4 predicts  $\alpha = 1.5$ . The depth factor  $D$  models the lumped effect of the depth of a bubble, and the effect of different excitation strengths of the bubbles. Bubbles that are submerged more will be attenuated more. We model the probability distribution  $P(D)$  of depth factors as  $P(D) = \frac{1}{\beta} D^{\frac{1}{\beta}-1}$  which is easily implemented by computing

$$D = \text{rnd}^\beta \tag{7}$$

where rnd is a uniformly distributed random number between 0 and 1, and  $\beta$  an adjustable exponent. Larger values of  $\beta$  result in a larger fraction of weakly sounding bubbles.

The rising pitch of bubbles formed close to the surface is modeled by the parameter  $\xi$  which is controlled by a slider. Since bubbles with a rising pitch are created close to the surface it seems reasonable to assume they are generally louder than average. This effect is modeled by the “riseCutoff” slider. When it is set to a value  $0 \leq x \leq 1$ , only bubbles with a depth factor  $D$ , as defined in Eq. 6, which is larger than  $x$  have a non-zero  $\xi$ .

The relative creation rates of the bubbles can be set with an individual slider for every bubble radius. Presets are defined for various distributions. A flat distribution results in equal creation rates for all bubbles. A  $1/r^\gamma$  distribution can be chosen, which results in relative rates of the form

$$\bar{\lambda}_k = 1/r^\gamma. \tag{8}$$

The exponent  $\gamma$  can be controlled through a slider.

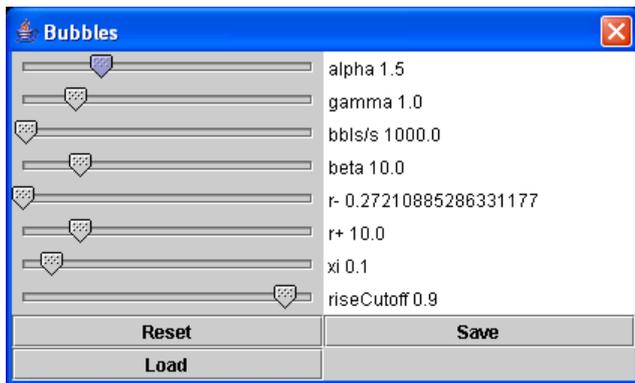


Figure 7: A slider based GUI allows the user to set bubble modeling parameters interactively.

## 6. RESULTS

An enormous variety of water-like sounds can be created with the simulator, ranging from intimate dripping sounds

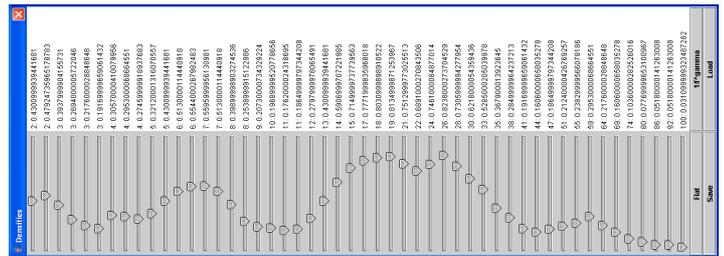


Figure 8: The density distribution over the bubble population can be set by adjusting each of the 50 sliders or by loading saved configurations from file. Flat or power distributions can be selected also.

to torrential rains or waterfalls. The resulting sounds are quite realistic, especially the more dense sounds. This is in part due to the nature of the stationary stochastic modeling, which is not justified for sparser sounds such as dripping and splashes.

Bubble creation rates of up to 1000/s produce sounds ranging from dripping to intimate streaming or light rain. Rates up to 10000/s produce sounds which are quite dense but still allow some individual droplets to be heard. Rates up to 100000/s produce blended soundscapes where individual droplets are not heard except occasionally, such as heavy rain or waterfalls. Yet higher bubble creation rates produce no appreciable difference anymore.

The effect of the exponent  $\alpha$  which models the energy injected into bubbles as a function of their size via Eq. 6 is to brighten or darken the sound. Smaller values of  $\alpha$ , which inject more energy into smaller bubbles, create the illusion of moving close to the water, whereas larger values create a feeling of distance. This is probably due to the fact that in nature the higher pitched components of water sounds are more attenuated by the environment at a distance than the low frequency components.

The exponent  $\beta$ , which models the sound attenuation of bubbles due to their depth in the water through Eq. 7, has an audible effect only for sparse sounds where the individual bubbles can be heard. A value of  $\beta = 10$  tends to sound best in most cases where there is an audible effect.

The bubble size distribution has a very strong effect on the resulting sound. For distributions of the form  $1/r^\gamma$  (see Eq. 8) values from  $\gamma = 0$  to  $\gamma = 3$  produce sounds of streaming whereas values above approximately  $\gamma = 5$  sound very much like rain. The effect of the range of radii of the bubble population has a very large effect. Removing the smaller bubbles from the synthesis creates a sound of blowing bubbles with a nozzle or straw, presumably because this mechanism of producing water sounds tends to create large bubbles, whereas all the other mechanisms always create a population which includes very small bubbles. Removing

the smallest bubbles from the population (.2mm) always reduces the quality of streaming and dripping sounds and seems to play an essential role. Extending the range of radii to include large bubbles up to 5cm creates either the effect of a nozzle blowing under water or, for large bubble creation rates, the effect of a waterfall or large river.

The rise factor  $\xi$  improves the realism of sparse sounds somewhat for riseCutoffs of about 0.9. Denser sounds up to 10000/s with a large  $\xi$  and zero riseCutoff (meaning all bubbles have a rising pitch) create the illusion of air being injected violently into water close to the surface. It can either sound as produced by a nozzle or by a small waterfall. Denser sounds with a large  $\xi$  value produce a flanging artifact and do not sound realistic.

The quantization of the bubble radii to 50 values is aurally satisfactory. Increasing this value does not produce any audible difference, but reducing it creates some audible artifacts.

## 7. CONCLUSIONS

A physically based model for the real-time synthesis of the sounds made by water has been constructed. The model is based on the acoustical properties of bubbles as predicted by the Minnaert model. Individual bubble sounds are reminiscent of the sounds of water drops for certain bubble radii as was verified with a small user study. In reality, almost all liquid sounds involve a population of bubbles. A real-time liquid sound synthesizer was constructed which uses statistical models to excite a population of bubbles to create a wide range of liquid sound-effects under real-time parametric control.

Future work should focus on integrating the water sound synthesis method with liquid animation algorithms that are currently under development using various strategies to compute the bubble formation process from physical principles, to arrive at a multimodal physically based animation of water.

The large number of parameters needed to describe a complex liquid sound makes this synthesis method potentially useful for auditory data mapping. For this a better understanding of the perceptually relevant parameters leading to the perception of water and how they relate to the physical parameters would be very useful. The liquid sound synthesizer presented here would be a very useful tool to perform studies of this kind.

Finally, there may be audible improvements resulting from refinements of the single bubble model. First, the excitation of a bubble is not completely impulsive but has a characteristic profile which may have important audible consequences. Second, the Minnaert model applies to radially oscillating spherical bubbles only. Especially larger bubbles often deviate substantially from being spherical. Taking this

into account might improve the quality of the synthetic liquid sounds.

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