MEASUREMENT OF HOROPTER AND ALLEYS IN AUDITORY SPACE

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ABSTRACT

For binocular visual space, it is a well-known phenomenon that the horopter curves are not always physically straight, and that the form of the curves depends on the distance. A similar phenomenon is also known for tactile space. We conducted fundamental experiments on sound localization with distance, and confirmed that the horopter curves in auditory space are not always physically straight either, and that the form of the curves depends on the distance between the subject and the sound sources. We also clarified that the form of the horopter for auditory space is not the same but relatively similar to that for visual space, and is quite different from that for tactile space.

The use of a virtual environmental display system enabled the construction of a psychophysical experimental system to measure subjectively straight lines running phenomenally parallel to the median plane, the parallel alley, and subjectively straight lines running phenomenally equidistant from the median plane, the equidistance alley, in binaural auditory space. Experiments were conducted using the constructed experimental system, to measure the parallel alley and the equidistance alley in binaural auditory space. The following results were confirmed:

(1) The parallel alley and the equidistance alley in auditory space in a virtual environment are not always straight in the physical sense, and their forms depend on the distance from the median plane.

(2) The parallel alley lies inside the equidistance alley.

These tendencies are the same as those in visual space and tactile space in the real world.

Moreover, employing sound intensity and binaural time differences as parameters, space perception models were constructed to explain the dependency of the curves of the horopter, the parallel alley, and the equidistance alley on distance in auditory space. Based on the results of the simulation performed using the models, it was confirmed that the models successfully explain the results obtained in our psychophysical experiments.

1. INTRODUCTION

The phenomena of subjectively straight lines such as Helmholtz's horopter, and the parallel alley and the equidistance alley are well known as the characteristics of space perception in binocular visual space. Helmholtz's horopter is explained on the basis of the following phenomenon: Assuming that the x-axis is the intersection of the horizontal plane at eye level and the median plane of the observer, and the y-axis is the intersection of the same horizontal plane and the frontoparallel plane passing through both eyes of the observer (points L and R), as shown in Figure 1, horizontal lines that appear to be parallel to the y-axis (subjectively frontoparallel line) are not always parallel in the physical sense[1]. It is known that the physical form of the perceived frontoparallel lines depends on the distance between the observer and the parallel lines on the x-axis, that is, the form coincides with the physically parallel lines at a certain distance, to become concave when the observer is at nearer distances and convex at farther distances. A similar phenomenon is also known for tactile space[2][3].

On the other hand, the parallel alley and the equidistance alley represent the concept of subjectively parallel lines to the median plane. In Figure 2, assuming that the x-axis is the intersection of the horizontal plane at eye level and the median plane of the observer, and the y-axis is the intersection of the same horizontal plane and the frontoparallel plane passing through both eyes of the observer (points L and R), a pair of lines, p, denotes the parallel alley and a pair of lines, d, denotes the equidistance alley. The parallel alley is a pair of perceived

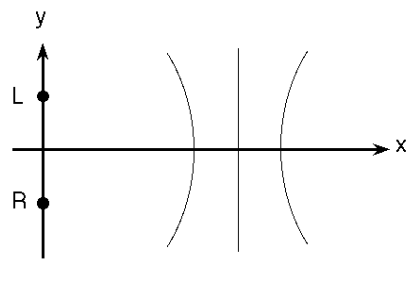

Figure 1. *Helmholtz's horopter*.

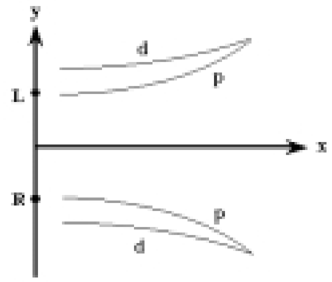

Figure 2. *Parallel alley and equidistance alley*.

straight lines obtained when two rows of luminous points on the horizontal plane at eye level are arranged symmetrically to appear straight and parallel to the median plane. Physically, the two lines should be straight and parallel to each other, but actually they are not; their form depends on the distance from the median plane (the distance from the x-axis). The equidistance alley is a pair of perceived straight lines obtained when two rows of luminous points on the horizontal plane at eye level are arranged symmetrically to appear straight and laterally equidistant from the median plane. Physically, the two lines should be straight and equidistant, but actually they are not; their form depends on the distance from the median plane[4]. The physical forms of these two types of subjectively straight lines have such a general tendency that the parallel alley lies inside the equidistance alley, as shown in Figure 2[5]. Concerning the disagreement between these subjectively straight lines and the physically straight lines, a similar phenomenon based on the sensation of upper limb movement has been observed for tactile space[2][6].

Based on these phenomena in visual space and in tactile space, we performed psychophysical experiments to examine similar phenomena in binaural auditory space.

2. MEASUREMENT OF THE HOROPTER IN AUDITORY SPACE

We conducted fundamental experiments on sound localization with distance, and confirmed a similar phenomenon to that of Helmholtz's horopter for perceived binaural auditory space.

2.1. Apparatus

Figure 3 is a diagram of the experimental system. The experiments were conducted in a completely darkened anechoic chamber. Three small loudspeakers (Foster Electric C040A001, 41-mm square, frequency bandwidth 500-7000 Hz) were placed at ear-level horizontal plane. Two of them were fixed symmetrically to the median plane of the subject, at a distance of 17-cm from either side of the median plane, and the third (the central speaker) could be moved closer or farther along the median plane. The default position of the central speaker was set at the intersection of the median plane and the line physically connecting the other two speakers.

The position of the central speaker and all signal timing were controlled by personal computer.

The head of each seated subject was fixed. The subjects used two push-button switches to register their responses.

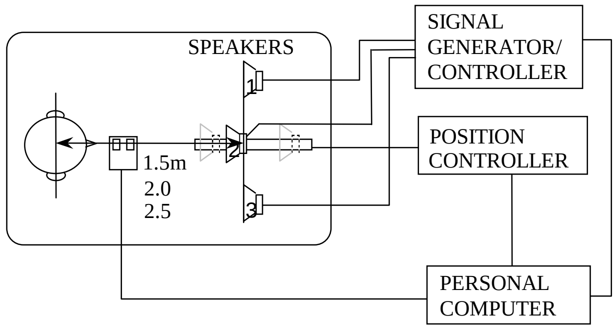

Figure 3. *Experimental system for auditory horopter*.

2.2. Method

We designed sound localization experiments using the method of constant stimuli[7]. In this method, the stimulus with a 50% probability of appearance in the obtained psychometric function was considered as the subjective equivalence when the method of two categories was employed. The purpose of the experiments was to determine the forms of perceptually straight horizontal lines in auditory space. Three small loudspeakers were used as sound sources to form a perceptually straight line. Subjects were asked to estimate the position of the central speaker relative to the other two. The basic stimulus consisted of a sequence of three, 200-ms bursts of white noise with 300-ms silence between the bursts. The noise was lowpassed with a filter having cutoff frequencies at 2800 Hz. The stimuli were transduced alternately by three speakers. The order of the transducing loudspeakers was reversed in each sequence with 1000-ms silence between the sequences. The overall level of the stimuli was approximately 45 dB in the case where the distance between the subject and the default position of the central speaker was 2.0 m.

The experiment was designed as follows. First, the central speaker moves to a certain position on the median plane, and sounds from the three speakers are provided sequentially to the subject until he/she perceives whether the position of the central speaker is farther than the imaginary straight line connecting the other two speakers. After the subject pushes one of two push-buttons corresponding to "farther" and "nearer," the sound stops and the central speaker moves to another position, and then sound is provided again, and so forth. The positions of the central speaker are selected from one of the following seven positions: -30 cm, -20 cm, -10 cm, 0 cm, 10 cm, 20 cm, and 30 cm away from the default position. Positive values indicate a position farther from the subject, and negative values are nearer. The order of the positions is selected randomly, while the total number of tests at each position is the same. The distances between the subject and the default position of the central speaker are 1.5 m, 2.0 m, and 2.5 m.

Eight adults (6 male, 2 female) served as volunteers. All were novice subjects. They ranged in age from 22 to 41 years.

2.3. Results

In order to estimate the perceived relative position of the central speaker statistically, the psychometric functions obtained from the experiments were analyzed using probit analysis[8]. Figure 4 shows typical examples of two types of resulting perceptually

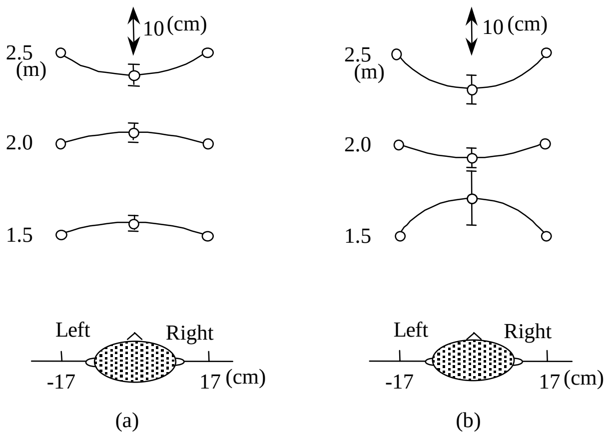

Figure 4. *Results of analysis*.

straight lines in auditory space. The midpoint of each line indicates the subjective position of the central speaker. This figure shows that horizontal lines that are perceptually straight on a subjective frontoparallel plane in auditory space are not always physically straight, and that their form depends on the distance between the subject and the sound sources; the curves are concave when the subject is at nearer distances and are convex at greater distances. When the estimated perceived position of the central speaker is equal to zero, the perceived straight line is also straight in the physical sense. We call the distance from the subject, which gives this particular straight line "the inflection point distance." In Figure 4(a), the inflection point distance of the auditory space is estimated to be between 2.0 m and 2.5 m. It is estimated to be between 1.5 m and 2.0 m in Figure 4(b).

When the distance between the subject and the speakers was 1.5 m, the following statistical hypothesis was tested: "the estimated perceived position of the central speaker is farther than the physically straight line connecting the other two speakers." The alternative hypothesis was rejected at a 0.05 % significance level for each subject. For a distance of 2.5 m, the hypothesis "estimated perceived position of the central speaker is nearer than the physically straight line" was tested. The alternative hypothesis was also rejected at the same level of significance for each subject.

2.4. Discussion

From the experimental results mentioned above, it was confirmed that the forms of the lines subjectively parallel to a frontoparallel plane are not always straight in the physical sense and depend on the distance between the sound source and the subject. The inflection point distance in auditory space is between 1.5 m and 2.5 m, which is not the same but relatively similar to that for visual space (less than 1.5 m). On the other hand, the form of the auditory horopter is different from that of tactile space which is concave to the subject regardless of distance (i.e., there is no inflection point distance).

3. MEASUREMENT OF THE PARALLEL ALLEY AND THE EQUIDISTANCE ALLEY IN AUDITORY SPACE

From the similarity of sensory modalities and the phenomenon of the auditory horopter, the phenomena of the parallel alley and the equidistance alley in auditory space are expected to be similar to those in visual space and tactile space. However, no studies have been conducted to verify this. In order to measure the form of the subjectively parallel lines in relation to the median plane in binaural auditory space, several sound sources should be located symmetrically with respect to the median plane as shown in Figure 5. In addition, the sound from each sound source should be output in order. However, such an experimental setup requires a large space, and the physical existence of nearer sound sources would occlude the sound from farther sound sources from reaching the ears of the subjects. These difficulties may prevent the successful performance of experiments for an auditory alley. Thus, the authors attempted to create a psychophysical experimental system to measure the forms of the parallel alley and the equidistance alley in auditory space by using the virtual environmental display system shown in Figure 6.

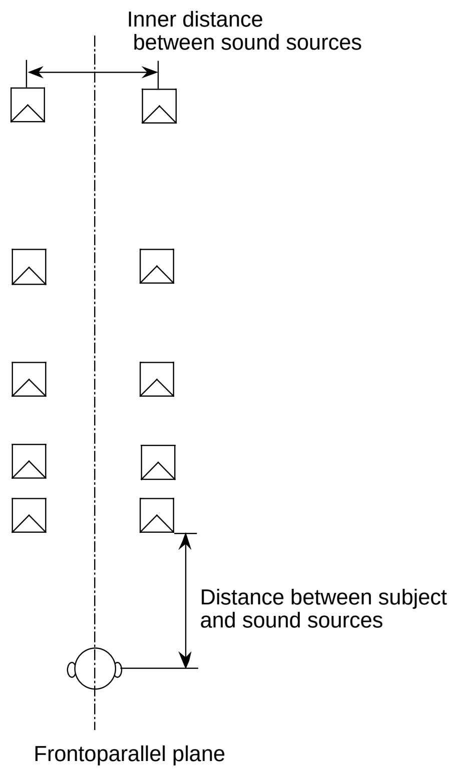

Figure 5. *Setup for experiments of auditory alley*.

Before a detailed description of our experiments, research on the auditory distance perception is briefly reviewed. As compared with the directions of sound sources, relatively little is known about the ability of human listeners to judge the distances of sound sources. According to the previous investigations[9][10][11][12][13], the mechanisms involved in auditory distance perception are complicated, and a number of different cues could theoretically be used to determine the distance of a sound source, such as intensity, binaural time difference, the ratio of early reflection from the room surface to direct sound, the decay of high frequency components by air absorption, and the familiarity of the sound stimuli to the listener. Among these acoustic cues, sound intensity seemed to be quite important for distance perception in the absence of reflections. Our previous study[14], in which the virtual environmental display system shown in Figure 6 was constructed and used, is one of the attempts which have been

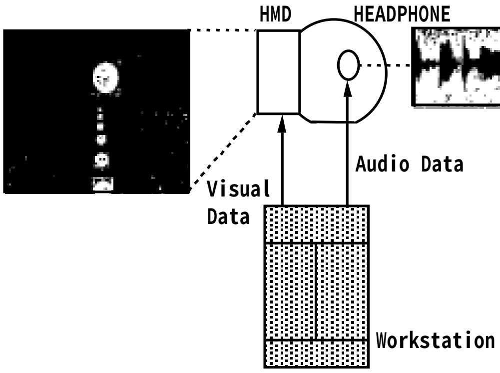

Figure 6. *Virtual environmental display system.*

made to clarify the relation between sound intensities and apparent distances from the sound sources quantitatively. The results were obtained from experiments carried out in a virtual environment; however, they successfully explained the results of the experiments conducted by M.B.Gardner[16], in a real environment. Our results under virtual environment conditions are also in good agreement with the results of the experiment conducted by G. von Békésy under real environment conditions [15] and with the results of the experiment conducted by D. R. Begault using intra-aural earphones[11].

3.1. Apparatus

The schematic of the virtual environmental display system used in this study is shown in Figure 6. This system is the same as the one used in our previous study[14] mentioned above. In order to construct the virtual environment, two workstations were used to generate computer graphics images for both eyes based on the virtual environmental model. These images served as the visual signals for the subject, which were sent through a calibrated see-through head-mounted display (STHMD) worn by the subject. The STHMD, in which the virtual environment can be superimposed on the real environment by a beam splitter, can be calibrated to make the coordinates of the visual space for a virtual environment coincide with those of the visual space for a real environment by the superimposition. A calibration method was developed and applied to the visual parameters of the STHMD to eliminate errors caused by mechanical misalignments in the STHMD[17] and by individual differences between the actual and designed locations of pupils of eyes, which ensured the coincidence of the apparent distance in the virtual environment with that in the real environment[18]. Using one of the computers, auditory signals were synthesized, output through DSP, and input through headphones (Senheiser HD25SP) to both ears of the subject. The output rate was set at 48kHz.

3.2. Method

We designed the experiments to measure the parallel alley and the equidistance alley in auditory space using the method of constant stimuli[7], that is, the same method was used as in the experiments to measure the horopter curves in auditory space. The schematics of the experimental systems used to measure the parallel alley and the equidistance alley are shown in Figure 7. A pair of sound sources symmetrically positioned with respect to the median plane of the subject is called a "pair of sound sources" hereafter. Four virtual sound sources are assumed, consisting of a standard pair of sound sources and a comparison pair of sound sources; the latter pair with various inner distances is located nearer to the subject than the former pair.

As visual signals, the images of two sound sources symmetrically positioned with respect to the median plane on the horizontal plane at ear level of the subject are displayed as the images of the standard pair of sound sources. The auditory signal given to both ears of the subject is pseudorandom noise, and its intensity and binaural time differences are controlled as follows:

Consider the sound stimuli that reach both ears of the subject from a sound source located at the left side of the median plane in Figure 5. For the sound stimuli that reach the left ear directly, the following notations are used:

P_l : The intensity of the sound stimuli that reach the left ear directly from the objective sound source;

d : The distance between the objective sound source and the subject;

P_0 : The standard sound intensity;

d_0 : The distance between the subject and the sound source from which the sound stimuli directly reach the left ear has the standard sound intensity.

As P_l is inversely proportional to the distance from the sound source[19][20], the intensity of the sound stimuli reaching the left ear of the subject is

$$P_l = \frac{d_0}{d} p_0 . \quad (1)$$

On the other hand, the sound stimuli reach the right ear via the circumference of the head of the subject, as shown in Figure 8. Therefore, the difference between the distance from the sound source to the right ear and the distance to the left ear (Δd) is approximated as follows, where the radius of the head of the subject is r [20]:

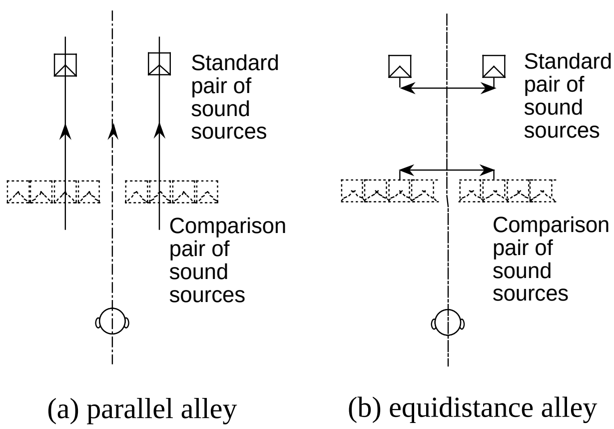

Figure 7. *Experimental system for parallel alley and equidistance alley.*

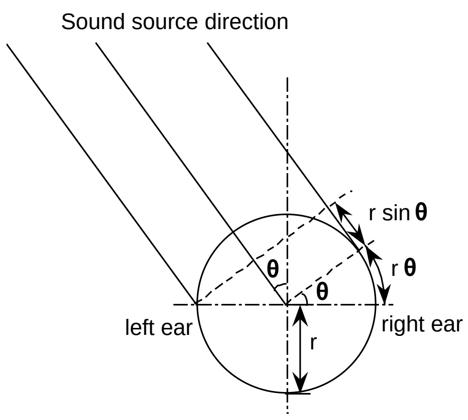

Figure 8. *Approximation of the difference between sound source for both ears.*

$$\Delta d = r\theta + r\sin\theta\ . \quad (2)$$

The angle θ is obtained using the distance from the sound source (y) and the distance between the sound source and the median plane (x):

$$\theta = \arcsin(x / y)\ . \quad (3)$$

Therefore, the intensity of the sound stimuli which reach the right ear (P_r) is given by

$$P_r = \frac{d_0}{d_0 + \Delta d}\, p_l\ . \quad (4)$$

Also, the time difference between the sound stimuli which reach the right ear and those which reach the left ear (Δt) is given as follows, where r is 8.75 cm and the sound velocity (v) is 340m/s:

$$\Delta t = \Delta d / v = 257(\theta + \sin\theta)\ [\mu\text{s}]. \quad (5)$$

The head of the subject is fixed and visual parameters of the STHMD are calibrated individually prior to conducting the experiments.

Each trial is carried out as follows:

(1) By assuming that the standard pair of virtual sound sources is located at a certain distance from both ears of the subject, parallel to the physical median plane and symmetric with regard to the plane, the sound stimuli supposed to be produced from the right source of the standard pair of sound sources and those supposed from the left are repeatedly given to the subject in order for a duration of 500ms at 500-ms intervals. In order to fix the subjective position of the standard pair of sound sources, the subject is instructed to move a pair of visual markers of the virtual sound sources to the position where they are perceived by using a mouse. Thereafter, the position of the pair of visual markers is fixed so that the subject can confirm the position of the standard pair of sound sources visually.

(2) Next, the sound stimuli assumed to originate from the standard pair and the comparison pair of sound sources are repeatedly given for a duration of 500 ms at 2500-ms intervals in order, and the subject is requested to input the result of his/her judgment using one of two mouse buttons. In the case of experiments for the parallel alley, the assumed order of the virtual sound sources of sound stimuli is the right standard sound source, the right comparison sound source, the left standard sound source, and the left comparison sound source, as shown in Figure 9(a). The subject judges whether the relationship between the line connecting the former two sound sources and the line connecting the latter two sound sources is type (b) or type (c), as shown in Figure 9. On the other hand, in the case of experiments for the equidistance alley, the assumed order of the virtual sound sources of sound stimuli is the right standard sound source, the left standard sound source, the right comparison sound source, and the left comparison sound source, as shown in Figure 10(a). The subject judges whether the inner distance of the latter two sound sources is narrower or wider than that of the former two sound sources, as shown in Figure 10(b) and 10(c).

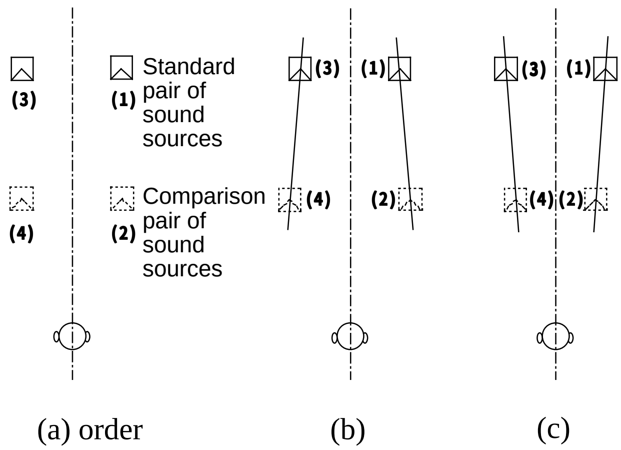

Figure 9. *Order of display and relations between two pairs of virtual sound sources in experiments on parallel alley.*

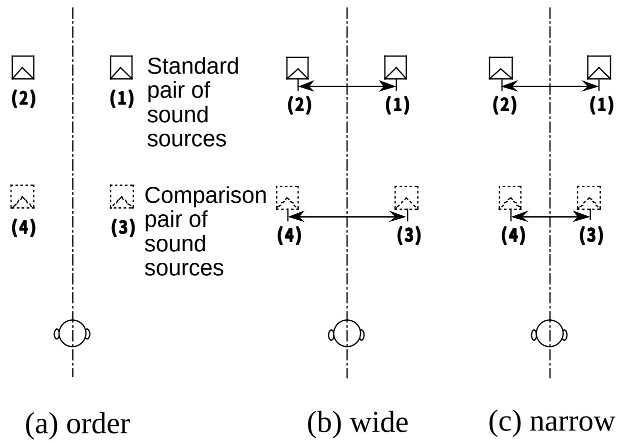

Figure 10. *Order of display and relations between two pairs of virtual sound sources in experiments on equidistance alley.*

The distance between a pair of sound sources is defined as the distance between the midpoint of the line connecting both sound sources of the pair, which is the intersection of the line and the median plane, and the midpoint of the line connecting both ears of the subject. The assumed distance and inner distance of the standard pair of sound sources are $4\sqrt{2}$ m, and

0.4 m respectively. The assumed distance of the comparison pair of sound sources is set as $2\sqrt{2}$ or 4 m, and their assumed inner distance is set as 0.0, 0.2, 0.4, 0.6, or 0.8 m.

3.3. Results

In one trial run of the experiments, five different inner distances of the comparison pair of sound sources were assumed four times in random order with a certain distance assumed for the pair. The relative position between the comparison pair and the standard pair of sound sources, or the width of the inner distance of the comparison pair was judged by the subject. Thus, the number of judgments for each trial was 20. For both the parallel alley and the equidistance alley, trials with two different assumed distances were conducted, that is, a set of experiments consisted of four trials. After confirming the phenomena by preparatory experiments, experiments for detailed data acquisition were conducted with three subjects.

As a result of each trial, psychometric functions were obtained, and probit analysis[8] was applied to estimate the equivalence of the inner distance of the comparison pair of sound sources, which was perceived to be parallel to or equidistant from the standard pair of sound sources. Figure 11 shows a typical example of the results. Open circles indicate the experimental results of the measurement of the parallel alley, solid circles indicate the experimental results of the measurement of the equidistance alley, and each central point indicates the estimated average of the distance of the comparison pair of sound sources from the median plane with standard deviation.

From the results of the analysis, the following general tendencies are observed:

(1) Both the parallel alley and the equidistance alley in auditory space have forms different from those of the physically parallel line to the median plane;
(2) The auditory parallel alley lies inside the auditory equidistance alley in physical space.

3.4. Discussion

As a result of the experiments conducted using a virtual environmental display system, it was confirmed that the forms of the subjectively straight lines parallel to the median plane (parallel alley) and the subjectively straight lines equidistant from the median plane (equidistance alley) in binaural auditory space are not always parallel in the physical sense, and depend on the distance from the subject. Moreover, it was found that the parallel alley lies inside the equidistance alley in auditory space. These tendencies are confirmed only under the virtual environment; however, they are similar to those for visual space or tactile space obtained in the experiments conducted in a real environment.

4. MATHEMATICAL MODEL FOR SPACE PERCEPTION TO EXPLAIN AUDITORY HOROPTER AND ALLEYS

Mathematical models to explain the horopter and alleys in auditory space were formulated. Based on the model, the results of the experiments described above were examined by simulation.

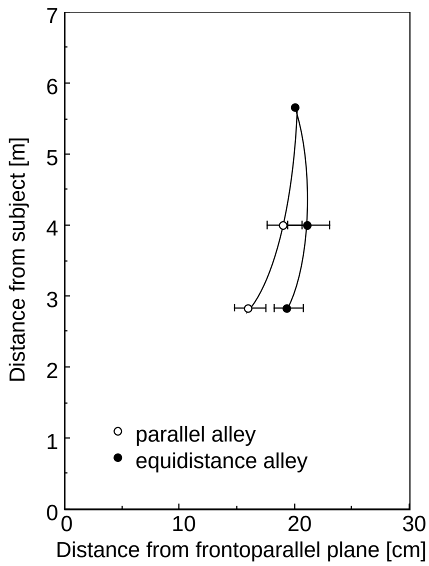

Figure 11. *Results of analysis*.

4.1. Formulation of the Mathematical Model for Space Perception

We have constructed a neural network model called the ISLES model (Independent Scalar Learning Element Summation Model) using biological information and constraints, and the model has successfully been used to explain the distance dependency of the forms of the horopter and alleys in visual space and in tactile space as a gap generated by the constraint of the learning process of space perception[21]. That is, in various types of space perception, invariant sensory information is set to be the standard function. Thus, the learning of the ISLES model, a neural network model with a scalar learning rule and with a constraint based on physiological knowledge, has a limitation which produces a specific error between the model and the standard function. This error causes the distance dependency of the forms of horopters and alleys in perceptional space.

As discussed in the previous section, according to various previous studies on auditory distance perception under real environment conditions and studies for auditory distance perception under virtual environment conditions[22][14], sound intensity and binaural time differences play an important role in auditory distance perception. Therefore, an attempt is made to construct an ISLES model to explain auditory horopter curves and auditory alley curves using sound intensity and binaural time differences as parameters. That is, assuming the ISLES model is a model for space perception to generate auditory alleys, sound intensity and binaural time difference are considered to be the input signals for the model.

Based on [23], sound intensity k is defined as the product of the sound intensity reaching the right ear and the sound intensity reaching the left ear. Then, the origin of a coordinate system in the physical space is set to be the midpoint of both ears, the x-axis is set to be the line passing through both ears, and the y-axis is set to be the horizontal line perpendicular to the x-axis. The sound intensity k of a certain position (x , y) in the coordinate system is represented as follows:

$$\left(x^2 + y^2 + a^2\right)^2 = 4a^2x^2 + \frac{I_S^2}{k}, \tag{6}$$

where I_S is the intensity of the sound source and a is the distance from the origin to both ears[23]. Also, for binaural time differences, Δt, the following equation is obtained from the equation of phase difference described in [23]:

$$\Delta t = \frac{\sqrt{(x+a)^2 + y^2} - \sqrt{(x-a)^2 + y^2}}{v}, \quad (7)$$

where v is sound velocity.

In a perceptional signal system for auditory space, in the case of learning to perceive the horopter curves, the target for the learning (the plane parallel to the frontoparallel plane) is physically described as

$$y = y_c, \quad (8)$$

where y_c is a constant. On the target plane for the learning, the equations to render each of the three variables, Δt, y, and k, invariant may be considered as candidates for the standard functions, where the fourth variable x is excluded because of its apparent variability.

As the standard equation for the horopter in auditory space, the following equation is assumed on the analogy of the standard equation for the horopter in visual space[21]:

$$H_h(k, \Delta t) = k + H_a(\Delta t), \quad (9)$$

where sound intensities(k) are compensated additionally by the function of binaural time differences (Δt), that is, k is considered invariant.

The ISLES model of Eq. (9) is assumed as follows:

$$\hat{H}_h(k, \Delta t) = k + \hat{H}_a(\Delta t). \quad (10)$$

The first term on the right-hand side, k, lies on the line parallel to the frontoparallel plane described by Δt. The second term, $\hat{H}_a(\Delta t)$, is the scalar function obtained from the convergence of the learning process for the perception of parallel to the frontoparallel plane.

Assuming the learning area for auditory space perception is from $y_{\min}$ to $y_{\max}$ in the direction of the distance from the frontoparallel plane (y), and the distribution of learning points is uniform with regard to y, the following equation is obtained by representing k by y and Δt:

$$\hat{H}_a(\Delta t) = -\frac{\int_{y_{\min}}^{y_{\max}} k(y,\ \Delta t)dy}{y_{\max} - y_{\min}}, \quad (11)$$

where the learning area of auditory space perception is from $y_{\min}$ to $y_{\max}$ in the direction of the distance from the frontoparallel plane(y).

That is, the constructed mathematical model to represent the horopter curves is the pair of Eqs. (10) and (11).

On the other hand, in the case of learning to perceive the parallel alley and the equidistance alley, the target for the learning (the plane parallel to and equidistant from the median plane) is physically described as

$$x = x_c, \quad (12)$$

where x_c is a constant. On the target plane for the learning, the equations to render each of the three variables, Δt, x, and k, invariant may be considered as candidates for the standard functions, where the fourth variable y is excluded because of its apparent variability.

As the standard equation for the parallel alley in auditory space, the following equation is assumed on the analogy of the standard equation for an auditory horopter, that is, Eq. (9):

$$H_p(k, \Delta t) = \Delta t + H_t(k), \quad (13)$$

where binaural time differences (Δt) are compensated additionally by the function of sound intensity (k), that is, Δt is considered invariant.

The ISLES model of Eq. (13) is assumed as follows:

$$\hat{H}_p(k, \Delta t) = \Delta t + \hat{H}_t(k). \quad (14)$$

The first term on the right-hand side, Δt, lies on the line parallel to the median plane described by k. The second term, $\hat{H}_t(k)$, is the scalar function obtained from the convergence of the learning process for the perception of parallel to the median plane.

Assuming the learning area for auditory space perception is from $x_{\min}$ to $x_{\max}$ in the direction of the distance from the median plane (x), and the distribution of learning points is uniform with regard to x, the following equation is obtained by representing Δt by x and k:

$$\hat{H}_t(k) = -\frac{\int_{x_{\min}}^{x_{\max}} \Delta t(x, k)dx}{x_{\max} - x_{\min}}, \quad (15)$$

where the learning area is from $x_{\min}$ to $x_{\max}$ in the direction of the distance from the median plane (x).

That is, the constructed mathematical model to represent the parallel alley is the pair of Eqs. (14) and (15).

With regard to the standard function for the equidistance alley $\hat{H}_d(k, \Delta t)$ in auditory space, "distance" should satisfy the additive law so that x can be considered invariant as follows:

$$H_d(k, \Delta t) = H_u(k) + H_c(\Delta t) = x_c. \quad (16)$$

Then the ISLES model of Eq. (16) is assumed as follows:

$$\hat{H}_d(k, \Delta t) = \hat{H}_u(k) + \hat{H}_c(\Delta t). \quad (17)$$

With regard to the first and second terms on the right-hand side, the learning area in the acquisition process for distance perception from the median plane is assumed from $y_{\min}$ to $y_{\max}$ in the direction of the distance from the frontoparallel plane passing through both ears (y), and the distribution of learning points is assumed to be uniform with regard to y. Then, the following equations are obtained by representing x by y and k, and by representing x by y and Δt, respectively, similar to Eq. (15):

$$\hat{H}_u(k) = \frac{\int_{y_{\min}}^{y_{\max}} x(y,\ k)dy}{y_{\max} - y_{\min}} \quad (18)$$

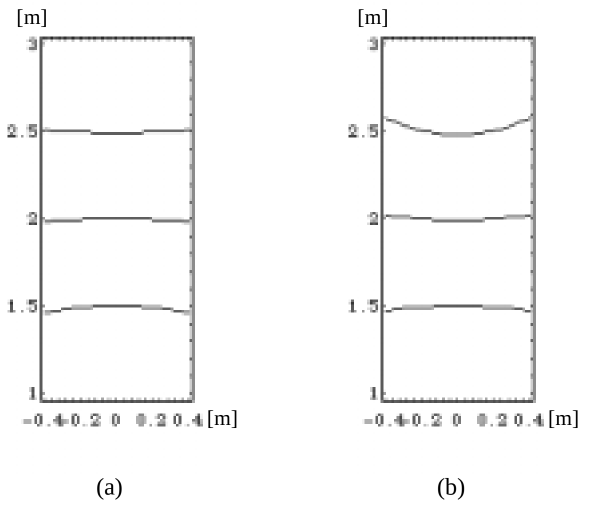

Figure 12. *Results of simulation for Eqs. (10) and (11).*

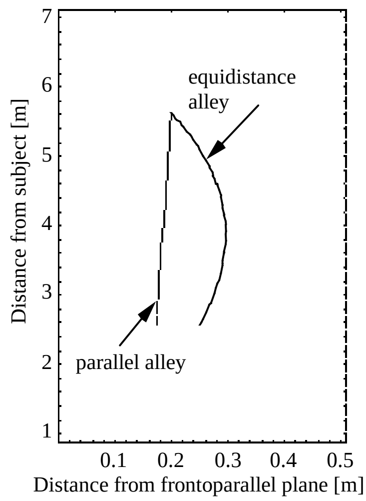

Figure 13. *Results of simulations for Eqs. (14) and (15) (parallel alley) and for Eqs. (17), (18) and (19) (equidistance alley) .*

$$\hat{H}_c(\Delta t) = \frac{\int_{y_{\min}}^{y_{\max}} x(y,\ \Delta t)dy}{y_{\max} - y_{\min}} \tag{19}$$

That is, the constructed mathematical model to represent the equidistance alley is the set of Eqs. (17), (18), and (19).

4.2. Examination of the Experimental Results

By executing Eq. (11) for various learning areas (areas for integration), the forms of Eq. (10) are examined by simulation. Examples of the results of simulation are shown in Figure 12, where $y_{\min}$ =1000[mm], $y_{\max}$ = 10000[mm] for (a) and $y_{\min}$ =1000[mm], $y_{\max}$ = 4000[mm] for (b).

By executing Eqs. (15), (18), and (19) for various learning areas (areas for integration), the forms of Eq. (14) (parallel array) and Eq. (17) (equidistance alley) are examined by simulation. Examples of the results of simulation for both curves through the standard pair of sound sources are shown in Figure 13 ($x_{\min}$ =0[mm], $x_{\max}$ = 300 [mm] for the parallel alley, and $y_{\min}$ =1000[mm], $y_{\max}$ = 18000[mm] for the equidistance alley).

From these results, it was confirmed, as a general tendency independent of the learning area, that these forms have tendencies similar to those seen in the experimental results for auditory alleys using a virtual environment, that is, the forms of the parallel alley and the equidistance alley are not always parallel to the median plane in the physical sense and the parallel alley lies inside the equidistance alley.
From the results, it was confirmed, as a general tendency independent of the learning area, that these forms have tendencies similar to those seen in the experimental results for auditory horopter and alleys.

5. CONCLUSIONS

We conducted fundamental experiments on sound localization with distance, and confirmed that the horopter curves in auditory space are not always physically straight, and that the form of the curves depends on the distance between the subject and the sound sources. We also clarified that the form of the horopter for auditory space is not the same but relatively similar to that for visual space, and is quite different from that for tactile space.

Using a virtual environmental display system, an experimental system to measure the parallel alley and the equidistance alley in auditory space was created. From the results of the experiments performed using the system, the following conclusions are drawn:

(1) The forms of the parallel alley and the equidistance alley in auditory space are not always straight in the physical sense, and depend on the distance from the median plane.
(2) The auditory parallel alley lies inside the auditory equidistance alley.

Although these results were obtained under a virtual environment condition, they show similar tendencies to those shown by the phenomena in binocular visual space and tactile space.

Employing sound intensity and binaural time differences as parameters, mathematical models were constructed to explain the horopter, the parallel alley, and the equidistance alley in binaural auditory space. From the results of the simulation, it was confirmed that the models successfully explain the results obtained from the psychophysical experiments.

As already mentioned in the first section, it is well known that the parallel alley lies inside the equidistance alley in visual space. Theoretical interpretation was first proposed by Luneburg[24]. Namely, this phenomenon is accounted for if visual space is assumed to be hyperbolic. If visual space is Euclidian, being parallel and being equidistant is the same, and the two alleys have to coincide in physical space also. If visual space is elliptic, it is expected that the parallel alley will lie outside the equidistance alley. Luneburg also gave a set of rationales for the assumption that visual space can be regarded as a Riemannian space of constant curvature: hyperbolic, Euclidean, or elliptic. Our experimental results obtained both from a real environment and a virtual environment may be interpreted by the non-Euclidean metric of binaural auditory space. They may also be taken into account for a practical approach to auditory display. We hope the experimental results and the constructed mathematical models to explain the results will represent a step forward in the psychophysics of hearing.

6. ACKNOWLEDGMENTS

We thank Mr. Sakurai for preparing the apparatus for fixing subjects, Ms. Izumi for participation as a subject, Mr. Oishi for supplying the STHMD, and other members of Tachi Laboratory, Graduate School of Information Science and Technology, The University of Tokyo for helpful discussion and advice.

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