SONIFICATION OF SPIN MODELS. LISTENING TO PHASE TRANSITIONS IN THE ISING AND POTTS MODEL.

K. Vogt, W. Plessas

Institute of Physics,
Department of Theoretical Physics,
University of Graz,
Universitätsplatz 5, 8010 Graz,
Austria
katharina.vogt@uni-graz.at
plessas@uni-graz.at

A. de Campo, C. Frauenberger, G. Eckel

Institute of Electronic Music and Acoustics,
University of Music and
Dramatic Arts Graz,
Inffeldg. 10/3, 8010 Graz, Austria
decampo@iem.at
frauenberger@iem.at
eckel@iem.at

ABSTRACT

In the interdisciplinary research project SonEnvir, we used sound to perceptualise data stemming from spin models. The advantages herein lie in the possibility of displaying more dimensions than in visual representation on one hand, and in the potential of the human auditory system on the other.

Spin models provide an interesting test case for sonification in physics, as they model complex systems that are dynamically evolving and not satisfactorily visualisable. While the theoretical background is largely understood, their phase transitions have been an interesting subject for studies for decades, and results in this field can be applied to many scientific domains. Also, most classical methods of solving spin models rely on mean values, whereas especially at the critical point of phase transition the fluctuations of single spins are their most important feature. We found that sound is an ideal display mode to study these fluctuations and the dynamic evolution of the whole model. Our sonifications allow for identifying the different phases easily, independent of the dimension of the model and the number of spin states. Also one gets a first idea about the order of the phase transition.

1. BACKGROUND

Sonification has been used in physics rather intuitively, without refering to the term explicitly. The classical examples are the Geiger counter and the Sonar, both monitoring devices for physical surroundings. An early example of research using sonification is the experiment of the inclined plane by Galileo Galilei. Following Drake [1], it seems plausible that Galilei used auditory information to verify the quadratic law of falling bodies (see figure 1). In reconstructing the experiment, Riess et al. [2] found that time measuring devices of the 17th century are less precise than auditory rhythm information.

Also in modern physics, sonification has already played a role: one example of audification is given in a paper by Pereverzev et al., where quantum oscillations between two weakly coupled reservoirs of superfluid helium 3 (predicted decades earlier) were found by listening: *Owing to vibration noise in the displacement transducer, an oscilloscope trace [...] exhibits no remarkable structure suggestive of the predicted quantum oscillations. But if the electrical output of the displacement transducer is amplified and connected to audio headphones, the listener makes a most remarkable observation. As the pressure across the array relaxes to zero there is a clearly distinguishable tone smoothly drifting from high to low frequency during the transient, which lasts for several seconds. This simple observation marks the discovery of coherent quantum oscillations between weakly coupled superfluids.* [3]

Figure 1: *Experimental device of Galileo Galilei for experiments of the law of falling bodies. In rolling down the inclined plane, the ball hits the bells which are attached following a quadratic law. The resulting rhythm is regular. This device is rebuilt at the Istituto e Museo di Storia della Scienza in Florence. ©Photo Franca Principe, IMSS, Florence.*

Next to sonification methods in physics, physics methods found their way into sonification, as in the model-based sonification approach by Hermann et al. [4]. In so called data sonograms, physical formalisms are used to explore high-dimensional data spaces.

1.1. Physics and sonification

Starting from psychoacoustics, the advantages of sonification in general are obvious, see [6] for a review. In physics, sonification has special advantages. First of all, modern particle physics is usually described in a four-dimensional framework. This makes it hard to visualise and thus very abstract - in didactics and research, sonification may help. In the auditory domain, many parameters may be used to display a four-dimensional space. Even if we handle a three dimensional space evolving in time, a complete visualisation is not possible any more. A feature of auditory dimensions that has to be taken into account is that these are generally not orthogonal, but could rather be compared to mathematical subspaces [7]. This concept is very common in physics, and thus easily applicable. Furthermore in physics, many phenomena are wave phenomena happening in time, just as sound is. Thus sonification provides a very direct mapping. While often scientific graphs map physical phenomena in the time direction, this is not necessary in a sonification, where the *physical* time persists, and more parameters may be displayed in parallel.

Of course, sonification can only be a complementary tool to classical analytical methods, but it may be a crucial one. We accept, for instance, visual interpretation in many scientific fields as an analysis tool, which is often superior to or at least preceding mathematical treatment. For instance, G. Marsaglia [8] described tests for the quality of numerical random number generators. One of these is the parking lot test, where mappings of randomly filled arrays in one plane are plotted and visually searched for regularities. In the description, he argues that visual tests are *striking*, but *not feasible in higher dimensions*. An all-encompassing mathematical test of this task cannot be provided. Sonification is a logical continuation of such analytical methods.

The major disadvantage of sonification we encountered is that physicists (as scientists in general) are not used to it. visualisation techniques and our learnt understanding of them has been refined since the very beginnings of modern science itself. For auditory perception especially, we were e.g. confronted with the idea of the hearing process being *just* a Fourier transformation. This example illustrates that still a lot of convincing has to be done.

1.2. SonEnvir project

In the research project SonEnvir, we addressed actual problems of different disciplines with the help of sonification. As SonEnvir is an interdisciplinary project, we profited from approaches of the other target sciences in the project, namely Sociology, Neurology and Signal Processing and Speech Communication. For more information on the project please refer to [5].

In the course of the project, we searched for applications of sonification within theoretical physics, especially particle physics and statistical physics. Many problems there are analytically well understood and exploit the data in a way of abstracting it. Thus details are often suppressed and an intuitive understanding cannot be given. This approach often aims at a visual exploitation of results, for instance by reducing the dimensions (e.g. a multi dimensional system can be mapped into one plane). With sonification, one has to start afresh and track the basics of the problem again. We decided to focus on statistical spin models of computational physics, for various reasons given below.

Other approaches to sonification in theoretical physics within SonEnvir project and with outside partners dealt with Baryon spectra of Constituent Quark Models ([10] and [11]) and sonifications of the Dirac spectrum, e.g. [12]. Smaller projects dealt with the chaotic double pendulum and quantum chromodynamics calculated on numerical lattices, see [5].

The rest of the paper is structured as follows: In the next section (2) we give an overview of the physical background of spin models, classical solving procedures and their computation. In section 3 we outline different features of spin models that were utilised in the sonifications and describe the different sonification tools and results. Results of a qualitative evaluation of experts in the field is given in section 4. A conclusion is given in section 5. The appendix lists short descriptions of all audio files which are provided at: *http://sonenvir.at/downloads/spinmodels/*.

2. SPIN MODELS

2.1. Introduction

Spin systems describe macroscopic properties of materials (e.g. ferro-magnetism) by computational models of simple microscopic interactions between single elements of the material. The principal idea of modeling spin systems is to study a complex system in a controlled way, where they are theoretically tractable and mirror the behaviour of real compounds.

On the theoretical side, these models are interesting because they allow studying the behaviour of universal properties in certain symmetry groups. This means that some properties do not depend on details like the kind of material, such as so-called order parameters giving the order of the phase transition. Already in 1945, E. A. Guggenheim (cited in [16]) found that the phase diagram of eight different fluids he studied shows the very same coexistence curve (this is true when plotted in so-called reduced variables, the reduced temperature being T/T_{crit}, the actual temperature referred to the critical one, likewise the pressure). A theoretical explanation is given by a classification in symmetry groups - all of these different fluids belonged to the same mathematical group.

Besides macroscopic observables, as the overall magnetisation, one is interested in the microscopic properties of the system. Therefore we started out with the fluctuations of the spins, and provided auditory information that can be analysed qualitatively. Our goal was to display three-dimensional dynamic systems, distinguish the different phases and study the order of the phase transition. Audification and sonification approaches should be implemented for spin models. Both real-time monitoring of the running model and analysis of pre-recorded data sets should be tested. An emphasis was laid on microscopic information, but also analytic data pre-processing was done.

2.2. Physical Background - Ising and Potts Model

One of the first spin models, the Ising model, was developed by Ernst Ising in 1924 in order to describe a ferromagnet. Since the development of computational methods, this model has become one of the best studied models in statistical physics, and has been extended in various ways.

Its interpretation as a ferromagnet involves a simplified notion of ferromagnetism.[1] As shown in figure 2 it is assumed, that the magnet consists of simple atoms on a quadratic (or in three dimensions cubic) lattice. At each lattice point an *"atom"* is located with a magnetic moment (a spin) up or down. In the computation, on the one hand, neighouring spins try to align to each other, which is

[1] There are many different application fields for systems with next-neighour interaction and random behaviour. Ising models have even been used to describe social systems, as e.g. in [15], though this is a disputed method in the field.

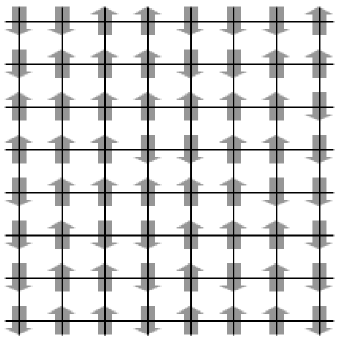

Figure 2: *Schema of Spin Models by the example of the Ising model with a lattice size of 8 times 8. At each lattice site, a spin can take two possible values (up or down).*

energetically more favorable. On the other hand, an overall temperature causes random spin flips. At a critical temperature T_{crit}, this process is undecided and there are clusters of spins on all orders of magnitude. If the temperature is lowered from T_{crit}, one spin orientation will prevail. (Which one is decided by the random initial setting.) Macroscopically, this is the magnetic phase ($T < T_{crit}$). At $T > T_{crit}$, the thermal fluctuations are too strong for uniform clusterings of spins. There is no macroscopic magnetisation, only thermal noise.

A straightforward generalisation of this model is the admission of more spin states than just up and down. This was realized by Renfrey B. Potts in 1952, and was accordingly called the Potts-Model. Several other extensions of models were studied in the past. We worked with the q-state Potts-Model and its special case for $q = 2$, the Ising model, both classical spin models.

In mathematical terms, the Hamilton-function H defines the overall energy, which any physical system will try to minimize:

$$H = -J \sum_{<i,j>} S_i S_j - \mathcal{M} \sum_i S_i \qquad (1)$$

where J is the coupling parameter between spin S_i and its neighbouring spin S_j. J is inversely proportional to the temperature; $\mathcal{M}$ is the field strength of an exterior magnetic field acting on each spin S_i. The first sum is denoted over nearest neighours and describes the coupling term. It is responsible for the phase transition. If $J = 0$, only the second term remains, and the Hamiltonian describes a paramagnet, being only magnetised in the presence of an exterior magnetic field. In our simulations, $\mathcal{M}$ was always 0.

When studying phase transitions macroscopically, the defining term is the free energy F.

$$F(T, H) = -k_B T ln Z(T, H) \qquad (2)$$

It is proportional to the logarithm of the so-called partition function Z of statistical physics, which sums up all possible spin configurations and weights them with a Boltzmann factor k_B. Energetically unfavorable states are less probable in the partition function than energetically favorable ones.

$$Z = \sum_{S_n} e^{-\frac{H}{k_B T}} \qquad (3)$$

The order of the phase transition is defined by a discontinuity in the derivates of the free energy (see figure 3). If there is a finite discontinuity in one of the first derivatives, the transition is called *first order*. If the first derivatives are continuous, but the second derivatives are discontinuous, it is a so-called *continuous phase transition*.

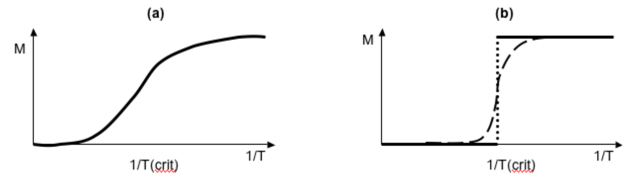

Figure 3: *Schema of the order of the phase transition. The mean magnetisation is plotted vs. decreasing temperature. (a) shows a continuous phase transition and (b) the phase transition of first order. In the latter, the function is discontinuous at the critical temperature. The roughly dotted line gives an approximation on a finite system, e.g. a computational model. The bigger the system, the better this approximation fits the discontinuous behaviour.*

It is a question of combinatorics to see that the partition function Z (eq. 3) is not calculable in practice: a three dimensional lattice with a length of 100 and two possible spin states has $2^{100^3} = (2^{10})^{10^5} \sim 10^{300.000}$ configurations that would have to be summed up - in each time step of the simulation. Also in an analytic deduction only few spin models have been solved exactly, and in three dimensions not even the simple Ising model is analytically solvable. Therefore classical treatment relies mainly on approximation methods, which allow partly to estimate critical exponents, and can be outlined briefly as follows:

Early theories addressing phase transitions, like Van der Waals theory of fluids and Weiss theory of magnetism can be subsumed under Landau theory or mean-field theory. Mean-field theory assumes a mean value for the free energy. Landau derived a theory, where the free energy is expanded as a power series in the order parameter, and only terms are included which are compatible with the symmetry of the system. The problem is that all of these approaches ignore fluctuations by relying only on mean values. (For a detailed review of phase transition theories please refer to [16].)

Renormalization group theory by K. G. Wilson [17] solved many problems of critical phenomena, most importantly the understanding of why continuous phase transitions fall into universality classes. The basic idea is to do a transformation that changes the scale of the system but not its partition function. Only at the critical point the properties of the system will not change under such a transformation, and it is then described by so-called fixed points in the parameter space of all Hamiltonians. This is why critical exponents are universal for different systems.

Nowadays, spin models are usually simulated with *Monte-Carlo algorithms*, giving the most probable system states in the partition function [16, p. 96]. We implemented a Monte Carlo simulation for an Ising and Potts model in SuperCollider3 (see figure 4). The lattice is represented as a torus (see fig. 7) and continually updated: for each lattice point, a different spin state is proposed, and the new overall energy calculated. As shown in equation 1, it depends on the neighour's interactions ($S_i S_j$) and the overall temperature (given by the coupling $J \sim 1/T$). If the new energy is smaller than the old one, the new state is accepted. If not, there is still certain chance that it is accepted, leading to random spin flips representing the overall temperature.

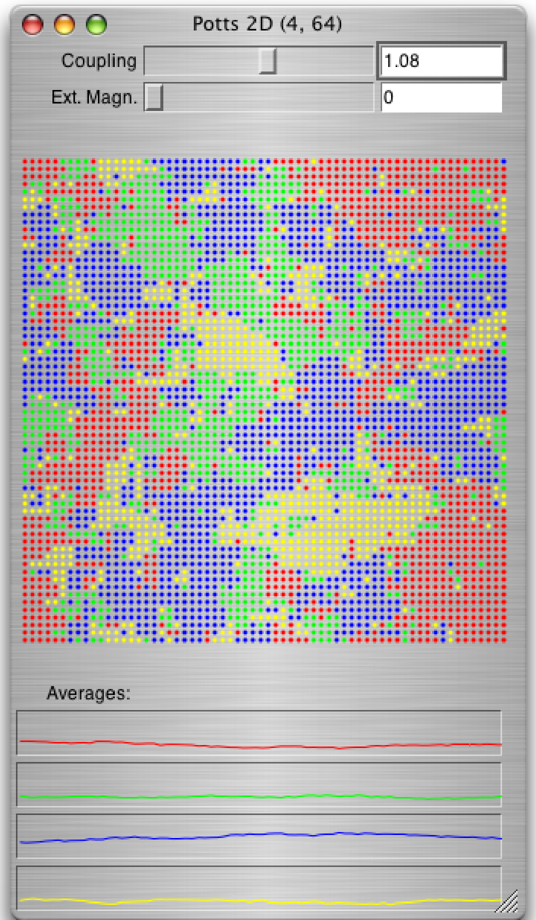

Figure 4: *Graphical user interface of the sonification tool for the 4-state Potts Model above critical temperature, where large clusters emerge. The lattice size is 64x64. The averages below the spin frame show the development of the mean magnetisation for the 4 spin parities over the last 50 configurations. As the temperature is constant and the system has been equilibrated before, these mean values are rather constant.*

To observe the model and draw conclusions from it, usually mean values of observables are calculated from the Monte-Carlo simulation, e.g. the overall magnetisation. The simulation needs time to equilibrate at each temperature in order to model physical reality, e.g. with small or large clusters. Big lattices with a length of e.g. 100 need many equilibration steps. With a typical evolution of the model, critical values or the order of the phase transition can be deduced. This is not rigorously doable, as on a finite lattice a function will never be discontinuous, compare figure 3. In a finite system, the *"jump"* in the observable will just *look more sudden* for a first order phase transition.

This last point is an argument for sonification and a first research goal for this work: in using more information than mean values, the order of the phase transition can be more clearly distinguished. Also, we studied different phase transitions with the hypothesis that there might be principal differences in the fluctuations, which can be better heard. (A Potts model with $q \leq 4$ states has a continuous phase transition, whereas with $q \geq 5$ states it has a phase transition of first order.) Thus researchers may gain a quick impression of the order of the phase transition.

In all analytical approaches above, the solving procedures of models are based on abstract mathematics. This gives great insight in the universal basics of critical phenomena, but often a quick glance on a graph complements classical analysis, as mentioned above. Thus in areas where visualisation cannot be done, we wanted to use sonification to help for an intuitive understanding without many underlying assumptions. Sonification tools can also serve as monitoring devices for highly complex and high dimensional simulations. The phases and the behaviour at the critical temperature can be observed. Finally, we were especially interested in sonification of the critical fluctuations with self-similar clusters on all orders of magnitude.

We wanted to allow for a more or less direct observation of data on all levels of the analysis to reassure assumptions and not overlook new insights. This should be done by observing the dynamic evolution of the spins, not mean values. Thus, the important characteristic of spin fluctuations can be studied and the entire system continuously observed.

3. SONIFICATION DESIGNS

3.1. Features of Spin Models' Data

Spin models have several basic characteristics, which were used in different sonification approaches. These properties refer to the structure of the model, the theoretical background and its interpretation and were exploited for the sonification in the following ways:

- The models are *discrete* in space by fixed lattice positions and these are filled with discrete valued spins. The data sets are rather big, in the order of a lattice size of 100 in two or three dimensions, and are dynamically evolving. Because of the modeling, the simulations are only correct on the statistical average, and many configurations have to be taken into account. Thus a time estimate has to be done for the sonification, for instance using the sonification design space map [14]. A single auditory event that displays a recognisable characteristic requires about 3 ms. Hence for the auditory display, a time-saving audification (see section 3.3) or the omission of spins (see section 3.2) was chosen.
- The models are calculated by *next-neighour interaction* aligning the spins on the one hand, and *random fluctuations* on the other. The next-neighour property was at least partially preserved in moving along a torus path or a Hilbert-curve through the frame, see fig. 7 (in approaches 3.3, 3.4 and 3.5). The random nature of the model was preserved by taking random elements for the sonification (see 3.2).
- There is a global *symmetry* in the spins, thus -in the absence of an exterior magnetic field- no spin orientation is preferred. This was mapped for the Ising model by choosing the octave for the two spin parities (see 3.2). In the audifications (3.3 and 3.4) every spin orientation is assigned a fixed value, and symmetry is preserved as the sound wave only depends on the relative difference between consecutive time (or lattice-) steps.

- At the critical point of phase transition, the clusters of spins become *self-similar* on all length scales. We tried to use this feature in order to generate a different sound quality at the point of phase transition. This allows a clear distinction between the two phases and the (third) different behaviour at the critical temperature itself (see section 3.5.)
- A straightforward choice for the sound mapping was to design the sonification such that it automatically generates a *noise sound* at $T >> T_{crit}$ in all approaches.

3.2. Granular Sonifications

In this approach, the data was pre-processed. Thus, the sound can be better controlled and is more convenient to listen to than an audification based approach. Also, more sophisticated considerations can be included in the sonification design.

In a *cloud sonification* we sonified each spin as a very short soundgrain, and played them slightly delayed within a short time frame. For a 32x32 lattice this is doable in one second, which leaves about 3 ms for each sound grain. One second is not as fast as one would like to go through the entire frame, but a trade-off with the fact that we still play all available information. For bigger lattices, this approach is too slow for practical use.

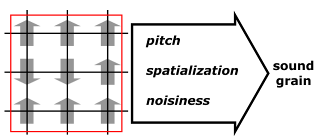

Figure 5: *Scheme of the sonification of random averaged spin blocks in the Ising model.*

Thus a similar approach was calculating mean values beforehand. We took *random averaged spin blocks* in the Ising model[2], compare fig. 5. The data was pre-processed for the sonification; we did not use all available information, but reduced the data points for the sonification to a random subset. At first, for each configuration a few lattice sites are chosen; then the average of their neighouring region is calculated, giving a mean magnetic moment between -1 (all negative) and $+1$ (all positive); 0 meaning the ratio of spins is exactly half/half. This information is used to determine the pitch and the noisiness of a sound grain. The more the spins are alike, the clearer the tone, the less alike, the noisier the sound. Spatial information is given by the location in space.[3] The soundgrains are very short and played quickly after one another from different virtual regions. With this setting, a three-dimensional *gestalt* of a cubic lattice is generated around and above the listener.

Without seeing the state of the model, a clear picture emerges from the granular sound texture, and also untrained listeners can easily distinguish the phases of the model. (Cf. audio files *IsingHot, IsingCold* and *IsingCritical*.)

[2] In this sonification we stayed with the simpler Ising model for realtime CPU power reasons, but the results do transfer to the Potts model.

[3] This feature of spatial hearing can only be properly reproduced with a multi-channel sound system. We adapted the settings for the CUBE, a multi-functional performance space with a permanent multi-channel system at the IEM Graz.

3.3. Audification Based Sonification

In this approach, we tried to utilise all possible information of the model. The basic idea was an audification, where the spins determine a waveform (see figure 6). The resulting sound wave can be listened to directly or taken as a modulator of a sine wave. When the temperature is lowered, regular clusters emerge, changing only slowly from time step to time step. Thus also in the audification longer structures will emerge, resulting in more tone-like sounds. When one spin dominates, there is no sound (except of some random thermal fluctuations at non-zero temperature).

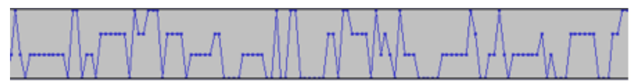

Figure 6: *Audification of a 4-state Potts model. The first 3 ms of the audio file of such a model with 4 different states in the high temperature phase (noise).*

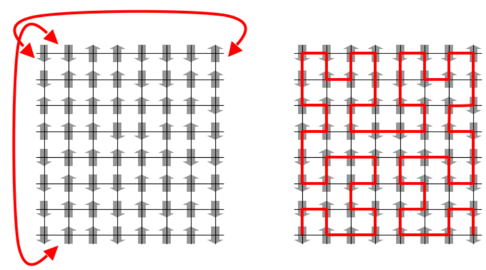

Figure 7: *Schemes of sequentialisations of the lattice used for the audification. The left scheme shows a torus sequentialisation, where spins at opposed borders are treated as neighours. This results in a torus - a doughnut shape, and row by row is read. On the right side a Hilbert curve is shown.*

While fig. 6 explains handling one line of data for the sonification, the question remains how to move through all of them. Different approaches of sequentialisation are shown in fig. 7. The program has periodic boundary conditions, so a *torus path* is possible. We also tried to go through the lattice on a *Hilbert curve*. This is a space (or room) filling curve for quadratic geometries, reaching every point without intersecting with itself. This prevents from wrong interpretation of different sounds, depending on whether rows or columns are read, especially in the case of symmetric clustering. It turned out that it is more preferable to use the torus path, as the model does in the calculation. Then every new data point can be used just after its calculation. Also, the symmetric clustering depends on unfavorable starting conditions and occurs only rarely.

Firstly, the sounds were recorded directly from the interactive model, using the GUI shown in fig. 4 for a specific temperature. In order to judge the phase of the system, this simple method is most efficient. Compare the files *NoiseA* and *NoiseB*, where a 3- and a 5-state model are run at high temperature, to the critical temperature in the 4-state model (*Critical*) and a value nearby (*Supercritical*) and to the equilibrated state at low temperature, where one spin already prevails (*SubCritical* recorded with the Ising model).

At the time of recording, the model has already been equilibrated - its state represents a typical physical configuration for the specific temperature. When the temperature is cooled down continually, the system needs several transition steps at each new temperature before the data represents the new physical state correctly. Thus, in a second approach, data was pre-recorded and stored as a sound-file. In contrary to our assumptions, the continuous phase transition is not very clearly distinguishable from the phase transition of first order. This is partly due to the data - on a finite lattice there are no discontinuous observables.

Partly, also the spins compensate each other at the critical point - as the spin parities cannot be distinguished from each other, one gradually rising wave form is masked by the others decreasing. Another conceptual problem is, that the equilibration steps (that are not recorded!) between the stored configurations cut out the meaningful transitions between them. Every lattice site is one sample (e.g. $32x32 = 1024$ lattice sites). With a sampling frequency of 44100 Hz, each 23 ms a completely different state is displayed, instead of perceiving a continuously evolving system with slowly changing cluster structures. This makes it more difficult to understand the dynamic evolution of the transitions. We tried to leave out as few equilibration steps as possible to stick close to a physical relevant state and still keep the transitions understandable. We recorded e.g. for a 32x32-lattice every 32^{nd} step, and on the whole 10 different couplings (temperatures), each 32 times. Thus, our soundfiles have (32 x 32) lattice sites x 10 couplings x 32 record steps = 327680 samples, and last 7,4 s.

Still, when comparing a 4-state Potts model to one with 5 spin states, the change in the audio pattern is slightly more sudden in the latter (Compare audio file *ContinuosTransition* to *FirstOrderTransition*).

3.4. Channel Sonification

We refined the audification approach of section 3.3, and allowed to record data for each spin separately. This concept is shown in figure 8. All of the lattice is sequentialised like a torus (see fig. 7) and read out as many times as there are numbers of spin states. When data of spin A is collected, only lattice sites with spin A are set to 1; all the others to 0. On the contrary, when spin B data is collected, all lattice sites with spin A are set to 0, and spin B to 1; and so forth.

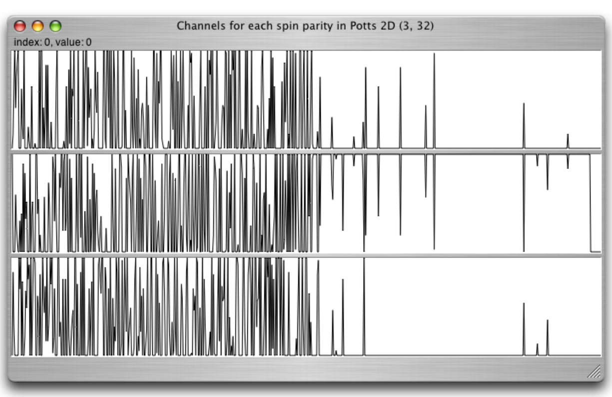

Figure 8: *The three recorded audio channels for a 3-state Potts model cooling down from super- to subcritical state.*

Thus, the different spins are separate and can played on different channels. The masking effects described in section 3.3 vanish. A problem is, that the channels are highly correlated: in the Ising model with only 2 states, the 2 channels are exactly reciprocal. Thus there is a possibly psycho-acoustical effect that makes it harder to distinguish the channels. Still, the overall impression is clearer than the simple audification, and this approach is the most promising regarding the order of the phase transition. (For the sound examples for this paper, the channels are panned to virtual sound sources in stereo, which makes them even harder to distinguish.)

3.5. Sonification of self similar structures

As a co-product of the above approach, we studied *self similar structures* at the point of phase transition by sonification. This may open a completely new research field, as self similarity is a visual next to mathematical concept, and its transfer in the auditory domain would allow a new point of view in science. The hypothesis that self similar structures may be audible was also strengthened by music, which exhibits self similar structures as well.

In a sonification and internal hearing tests we tried to display structures on several orders of magnitude in parallel. These were calculated by a blockspin transformation, which gives essentially the majority of one spin orientation in a region of the lattice. It was our goal to make such structures of different orders of magnitude recognisable as similarly moving melodies, or as a unique sound stream with a special sound quality.

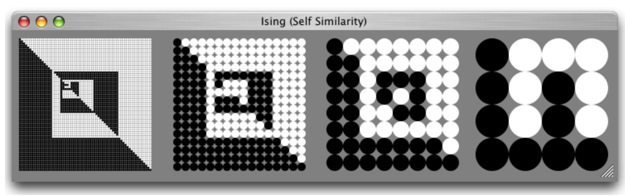

Figure 9: *A self similar structure of the Ising model as a testing case for self similarity. Blockspins are determined by the majority of spins of a certain region.*

In our approach, three orders of magnitude in the Ising model were compared to each other, as shown in figure 9. The whole lattice (on the right side - with the least resolved blockspins) was displayed in the same time as a quarter of the middle and as an eighth of the left blockspin spin structure (second on the left side). The original spins are shown on the left. Comparing three simultaneous streams for similarities in melodic behaviour has turned out to be a demanding cognitive task. We also experimented with a different approach: 3 streams representing different orders of magnitude are interleaved quickly, with brief pauses between them. When the streams are self-similar, one cannot hear a triple grouping; as soon as one stream is recognisably different from the others, a triple rhythm appears. While this method works with simple test data as shown in fig. 9, it does not convince with noisy data of running spin models.

4. DISCUSSION

A hearing test with statistical analysis was not appropriate as there are not enough subjects familiar with researching spin models. Thus, as a sort of qualitative evaluation we obtained opinions from experts in the field. These were four professors of Theoretical Physics in Graz, who were not directly involved in the sonifications. They were explained the results and given a few questions on the applicability and usefulness of the results described in this paper.

The overall attitude may be summed up as curious but rather sceptic, even if the opinions diferred in the details. Asked whether they themselves would use the sonifications, all of them answered to do so only for didactic reasons or popular scientific talks. The possibility of identifying different phases was acknowledged but was not seen superior to other methods (e.g. studying graphs of observables, as would be the standard procedure). One subject remarked that, for research purposes, the aha-moment was missing. This might be due to the fact that the Ising and Potts model have both been studied for decades and are well understood. While the data is mainly thermal noise, there is only few information to extract: Which phase, out of two possibilities and the transition? Which order of phase transitions, again out of only two possibilities? Our sonifications try a new approach and build up know-how for the sonification of more complicated models, but reveal no new physical findings. A three dimensional display seems interesting for them, even if the dimensions are not experienced explicitly (in the audification approach there is a sequentialisation for displaying one dimension) and the sound grain approach is limited to three physical dimensions.

Another application that was discussed is a quick overview over large data sets: e.g. checking numerical parameters (that there are enough equilibration steps, for instance) or getting a first impression of the order of the phase transition. This seems plausible to all subjects, even if the standard procedure, e.g. a program for pattern recognition, would still be equivalent and - given the familiarity with such tools - preferable to them.

The main point of criticism was the idea of a qualitative rather than quantifiable approach towards physics, which is seen as a possible didactics tool but not hard science. General sonification problems were discussed as well: it was remarked, that visualisation techniques play a more and more important role in science, and that they are hard competitors. Also for state of the art of publishing, sonification is at a disadvantage.

Besides this expected scepticism, it can be remarked that all subjects heard immediately the differences in the sound qualities. Metaphors to the sounds came up spontaneously during the introduction, as e.g. boiling water for the point of phase transition. The experts came up with several ideas for future projects to discuss; this kind of interest is an encouraging form of feedback.

5. CONCLUSION

This paper has presented sonification designs for spin models. Data was taken from Monte-Carlo simulations of Potts and Ising models implemented in SuperCollider3. These provide an interesting test case as they produce dynamically evolving data with their main characteristics being fluctuations of single spins. Although analytically well defined, finite computational models can only reproduce a numerical approximation of the predicted behaviour, which has to be interpreted.

A number of different sonifications were designed in order to study different aspects of spin models. We created a tool for the perceptualisation of lattice calculations, extendable to higher dimensions and a higher number of states. On the one hand, running models can be observed. On the other hand, pre-recorded data can be analysed in a way to get a first impression of the order of the phase transition.

We used different sonification techniques: Sound grain sonification of pre-processed data gives a reliable classification of the phase, the system is in, and allows to observe running simulations. It uses the random behaviour of spin models. Audification based tools allow us to make use of all the available data, and even track each spin orientation separately but in parallel. This tool is used to study the order of the phase transition. Additionally we worked on sonifications of self similar structures.

With this study, sonification has proven to be a complementary data analysis method for statistical physics. In the future we would like to enhance data quality and make different input models possible. We work on classification tools for phase transitions that allow to extend the dimensionality of the model further. Finally, we intend to apply the results to current research questions in the field of computational physics.

6. ACKNOWLEDGEMENTS

We would like to thank the SonEnvir-Team, notably to C. Dayé, G. Eckel and A. Wallisch. Also members of the Science By Ear Workshop in Graz in April 2006 shall be mentioned, who elaborated a sonification of the Ising model. These were (beside the authors) T. Hermann, H. Markum, J. Rohrhuber and T. Stockman. On the side of Theoretical Physics I would like to thank Chr. Gattringer, C. B. Lang, L. Mathelitsch and U. Hohenester. SonEnvir was funded by the Zukunftsfond Steiermark.

7. APPENDIX

The following audio files can be downloaded from *http://sonenvir.at/downloads/spinmodels/*.

The first part describes sonifications that enable the listener to classify the phase of the model (sub-critical, critical, supercritical).

- Granular sonification approach:

  Random, averaged spin blocks were used to determine the sound grains. The spatial setting cannot be reproduced in this recording. But even without having a clear *gestalt* of the system, the different characteristics of *IsingHot*, *IsingCritical* and *IsingCold* may easily be distinguished.

- Audification based approach:

  (Please consider that a few clicks in the audio files below are artifacts of the data management and buffering in the computer.)

  1. Noise: *NoiseA* gives the audification of a 3-state Potts model at thermal noise (coupling J = 0.4)

     NoiseB gives the same for the 5-state Potts model (J = 0.4), evidently the sound becomes smoother the more states are possible, but its overall character stays the same.

2. Critical behaviour: this example was recorded with a 4-state Potts model at and near the critical temperature:

   SuperCritical - near the critical point clusters emerge. These are rather big but homogeneous, hence a regularity is still perceivable. (J = 0.95)

   Critical - at the critical point itself, clusters of all orders of magnitude emerge, thus the sound is much more unstable and less pleasant. (J = 1.05)

3. *SubCritical* - as soon as the system is equilibrated in the subcritical domain (at $T < T_{crit}$), one spin orientation predominates, and only few random spin flips occur due to thermal fluctuations. (Recorded with the Ising model at J = 1.3.)

The next examples study the order of the phase transition.

- The direct audification gives only a very subtle difference between the two types of phase transitions:

  1. The 4-state Potts model is played in *ContinousTransition*.
  2. A more sudden change can be perceived in *FirstOrderTransition* for the 5-state Potts model.

- Audification with separated spin channels:

  For each spin-orientation the lattice is sequentialised and the resulting audification is played on an own channel. The lattice size was 32x32, and the system was equilibrated at each step. The examples finish with one spin orientation prevailing, which means that only random clicks from a non-vanishing temperature remain.

  1. The transition in the 2-state Ising model and the 4-state Potts model are continuous, the change is smooth.
  2. In the 5-state and 8-state models the phase transition is abrupt (the data is more distinct the more states are involved)

8. REFERENCES

[1] S. Drake, *Galileo*, Oxford University Press, New York, 1980.

[2] F. Riess, P. Heering, D. Nawrath, "Reconstructing Galileos Inclined Plane Experiments for Teaching Purposes", in *Proc. of the International History, Philosophy, Sociology and Science Teaching Conference*, Leeds, 2005.

[3] S. V. Pereverzev et al., "Quantum Oscillations between two weakly coupled reservoirs of superfluid 3He," *Nature*, Vol. 388, pp. 449 - 451, July 1997.

[4] T. Hermann, H. Ritter, "Listen to your Data: Model-Based Sonification for Data Analysis", in *Proceedings of the ICAD*, 1999.

[5] SonEnvir homepage, http://sonenvir.at.

[6] G. Kramer (Ed.), "Auditory display, Sonification, Audification and Auditory Interfaces", in *Proc. Vol. XVIII, Santa Fe Institute*, 1994.

[7] A.J. Hollander, *An Exploration of Virtual Auditory Shape Perception*, masters thesis, Univ. of Washington, 1994 (http://www.hitl.washington.edu/publications/hollander/).

[8] G. Marsaglia, "DIEHARD: A Battery of Tests for Random Number Generators", http://stat.fsu.edu/ geo/diehard.html.

[9] *SuperCollider3*: The programs for this project were written in *SuperCollider3*, a programming language and engine f/or real time audio synthesis, originally written by James McCartney and now under the free GNU license. It is available for download at: http://supercollider. sourceforge.net.

[10] A. de Campo, C. Frauenberger, R. Höldrich, T. Melde, W. Plessas, B. Sengl, "Sonification of quantum spectra", in *Proc. of the 11th International Conference on Auditory Display*, Limerick University, Ireland, 2005.

[11] de Campo A. , Dayé C. , Frauenberger C. , Vogt K. , Wallisch A. , Eckel G., "Sonification as an interdisciplinary working process", in *Proc. of the 12th International Conference on Auditory Display*, London, UK, 2006.

[12] A. de Campo, N. Hörmann, H. Markum, W. Plessas, K. Vogt, "Sonification of Monopoles and Chaos in QCD", in *Proc. of ICHEP'06 - the XXXIIIrd International Conference on High Energy Physics*, Moscow, Russia, 2006.

[13] B. Arons, "A review of the cocktail party effect," *Journal of the American Voice I/O Society*, 12, 35-50, 1992.

[14] de Campo A., "A data sonification design space map", in *Proc. of the 2nd International Workshop on Interactive Sonification*, York, UK, February, 2007.

[15] P. Fronczak, A. Fronczak, J. A. Holyst, "Ferromagnetic fluid as a model of social impact", in *International Journal of Modern Physics C*, Vol. 17, No. 8, 1227-1235, 2006.

[16] J. M. Yeomans, *Statistical Mechanics of Phase Transitions*, Oxford University Press, 1992.

[17] Wilson, K G. and Kogut J. (1974). Physics Reports, 12, 75.