EDUCATIONAL SONIFICATION EXERCISES: PATHWAYS FOR MATHEMATICS AND MUSICAL ACHIEVEMENT

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ABSTRACT

This paper reports developments in the use of sonifications and sonification software for educational purposes. Adolescent subjects received training in Cartesian graphing over several sessions with sonification software and a sonification-enhanced curriculum. The project attracted students with low linguistic and logical-mathematical capabilities. Students were engaged by musical composition activities, but they remained anxious about traditional mathematics activities. Though students' mathematical abilities improved only slightly *according to a traditional mathematical assessment*, this project demonstrated the students' increased comfort level with the subject of mathematics and an increased understanding of the concepts within their own set of linguistics.

1. INTRODUCTION

The educational potential for sonification exercises, as demonstrated by student motivation and cognitive development, has previously been reported [1]. This work reported a need for increased interactivity in the software interface and improved curricula for use with the software. Without familiarity of the relevant principles in cognitive psychology, mathematics & music education, and educational technology, practical introduction of sonifications into the educational environment may be delayed and/or ineffective.

Cognitive psychology offers many models for effective teaching practices. Here we find research that supports multimodal mathematics education. Social constructivists understand that students build context-dependent conceptions based on previous social and cultural experiences [2]. Natural proclivity in mathematics, therefore, may actually be hindered if the pedagogical style does not match the student's intuitive knowledge structure [3]. The Cognitive Flexibility theory [4] advocates skill with multiple representations of concepts to assist with solving unique, complicated problems of the real world as they are confronted. The development of a flexible knowledge structure requires presentation of concepts in multiple representations. Further, Appelbaum [5] warns of a "desensitizing" trend in mathematics. By favoring limited modalities of representation in mathematics we risk the loss of perceptual abilities--our link to the natural phenomena we wish to understand. Applebaum tells us that mathematics' purpose is not to quantify the world, but to mediate our discovery of it. Applications mediated by an 'aural mathematics" may be unimaginable with traditional mathematics.

The auditory representation of numbers is rarely discussed in mathematics literature, whereas research based on visual representations is plentiful and are presented as analogs to the obstacles and opportunities facing users of non-traditional mathematical representations. Visual mathematicians gain credibility by emphasizing the merits of mental flexibility, regardless of representation style, in a knowledge framework [6][7]. Mathematics education researchers also point out that, historically, mathematics representations conventions are arbitrary and prone to change [8][9], therefore "richer representations" [10] of understanding rise as a measure of student success. Though visual representations have their merits, they are avoided for the following reasons: 1) cognitive difficulty transferring a non-visual concept to a visual representation 2) difficulty with teaching some concepts visually, and 3) socio/cultural biases in definitions of what constitute mathematics processes and abilities when teachers and students evaluate mathematical achievement [11].

Mathematics education and music education share several objectives (e.g. composition and algebraic problem solving both involve the organization of meaningful expressions using representative symbols). The problem solving strategies used by children to reproduce melodies (temporal order, comparison, inferring) and compose melodies (perception of musical structure task, and relation of structure to individual definition of music) are described by Barret [12] and DeLorenzo [13], respectively. Conant's [14] lists of computer composition software features that help students write melodies and accompaniments include: instantaneous composition monitoring, straightforward editing capability, and simultaneous visual and auditory representations. While this research discusses pathways to musically develop critical thinking and creative manipulation of notation (skills valuable to mathematicians), transfer of these abilities to mathematics is not straightforward.

Educational technology research unveils the "Best Practices" that are common knowledge to educators, but may be unfamiliar to sonification software developers and users. Classrooms must serve the student's need for context through authentic and challenging exercises. This means technology should be used to solve difficult problems and educational software should be valuable outside the educational environment [15]. Projects undertaken over several class periods are one way of invoking meaningful understanding and reflection [16]. Clearly stated learning objectives are important since students and teachers may become distracted and overwhelmed by the variety of products students can create [17].

2. OBJECTIVES AND METHODS

The desired learning outcomes of this work are for adolescent participants to be able to interpret, analyze, and create Cartesian graphs of numerical data. The "Sound Grid" software described in Section 3 is designed to meet this goal. The software was used in accord with the curriculum described in Section 4. The author administered the curriculum over the course of a month

in 3-6 sessions of 30-60 minute length apiece. Sessions were conducted individually with the author at the participant's school during school hours.

Participants were selected from alternative schools in Portland, Oregon, USA. Alternative schools serve students who have not fit traditional classroom settings. Losses of family members, pregnancy, learning disabilities, and behavior problems (including criminal activity) are some reasons for enrollment at alternatives schools. To some degree, the participants in this study have demonstrated the failure of traditional classrooms to serve their educational needs. Further, participation in the study was voluntary. In describing the project to participants the mathematical and musical activities were emphasized and led to some selection based on this interest. Each participant's performance on a standardized test prior to commencement of instruction demonstrated that no subject's mathematic ability was worthy of exclusion from participation. Participants were between the ages of 13 and 20. Three female and ten male participants began the study. During the course of the instruction 2 participants ceased attending school and 2 choose not to continue participation.

A 25 question standardized test on Cartesian graphs and algebraic equations was administered before and after the education sessions. The test was composed of sample questions from the GED mathematics test and the Oregon State Benchmark Level 3 (8th year) mathematics test. Participants were instructed to answer as best they could, to skip questions unfamiliar to them, but to answer challenging questions with which they had some familiarity. The following are examples of questions given to participants:
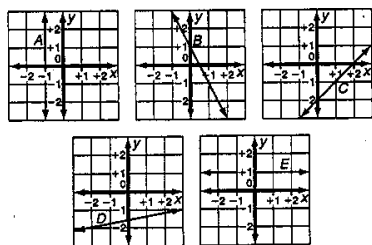
A. Find the slope of the line that passes through each pair of points.
(1,-3) and (0,1)   (4,5) and (3,-4)

B. The ordered pair (0,-2) is a solution of which of the following equations?
(1)   2x – 3y = -6
(2)   2x + 3y = 6
(3)   –2x + 3y = -6
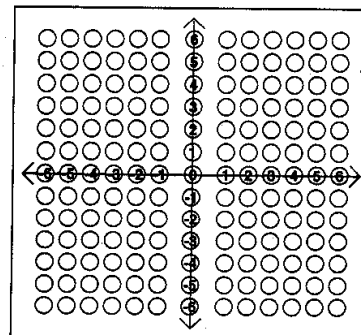*(4)   –2x – 3y = 5*
(5)   x + 2y = 4

C.

Which of the lines shown above has a negative slope?

(1) *A*
(2) *B*
(3) *C*
(4) *D*
(5) *E*

D.

Show the location of the point whose coordinates are (−3, 2).

To quantify the learning style preferences of each participant a multiple intelligences(MI) test developed by Howard Gardner, C.A. Armstrong, and the Boulder Center of Accelerated Learning (modified by Nancy Faris and the author) was also administered prior to the treatment. The test consisted of seven statements for each of the seven intelligences initially identified by Gardner: linguistic, logical-mathematical, musical, spatial, kinesthetic, interpersonal, and intrapersonal. Participants were instructed to select statements that described themselves.

To capture changes in participant thinking, concepts, and representational schemes, interviews were conducted before, during, and after the treatment. Instruction sessions and interviews were tape recorded and transcribed in their entirety. The following are examples of questions presented to all participants that provided the framework for in-depth questioning personalized to each participant:

Before:
What do you like and dislike about math?
What is your definition of mathematics?

During:
Have you been thinking about what we've learned last time?
Has what we've been doing affected how you think about the music you listen to?

After:
What did you like most about our lessons and the software?
How was this lesson different than other math lessons you've had?

3.   SOFTWARE DESIGN

The sonification software used for this study was designed with three principles in mind: 1) Interactivity—instantaneous playback, user-manipulation of sound parameters, and editing access to underlying data sets 2) Multiple linked representations of data displayed simultaneously [8] & 3) Application as a musical instrument. The application runs on the portable Java platform and utilizes functions of the JavaSound v.1.3.1 packages. A screenshot of the application, the "Sound Grid", is shown in Figure 1, and some basic features follow.
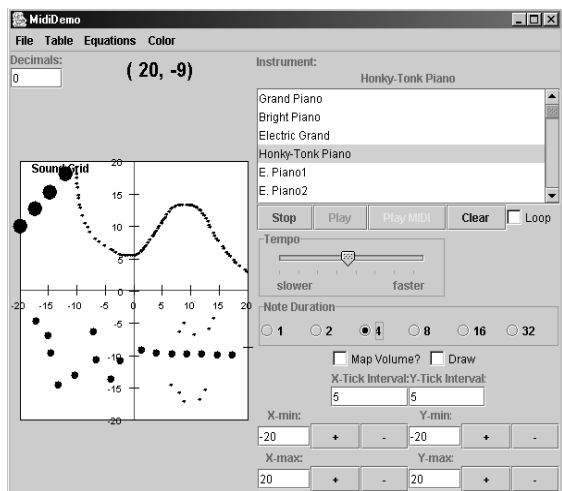
Figure 1. *Screenshot of the Sound Grid main window.*

The central feature of the application is an active Cartesian grid generating sound with cursor movement within the grid. Users can adjust the grid scales and tick mark intervals via input boxes. Single coordinate points may be added to the grid by either a mouse click, or as continuous discrete points that follow the mouse while in "Draw" mode. The "Coordinates" text updates the coordinate pair of the cursor as it moves in the grid. Each coordinate point is mapped to pitch on the vertical scale and volume is optionally mapped to the horizontal scale. Data points are stored in a vector that may be viewed in tabular form from the "Table" menu item. Data points in the table are updated and displayed in real time as coordinate points are added. The complete data set may be heard by clicking "Play" and the tempo of playback is adjustable. Points not appearing in the grid during playback are not heard. Several note durations are available, editable, and represented in the grid as points of increasing size with longer duration. Users can choose voices in the "Instrument" scroll menu. Tabular cut and paste functions are available for data transfer to and from spreadsheets. A Java Archive (JAR) file of the application and instructions for installation is found at http://web.pdx.edu/~psu25784/SoundGrid/SoundGrid.html.

4. CURRICULUM DESIGN

The curriculum for use with the Sound Grid was aimed at meeting state and national mathematics benchmarks in the area of Cartesian graphing. In a synthesis of progressive mathematics and music education curricula, four categories of educational activities were designed, as briefly outlined below. These activities attempt to inspire creativity in mathematical thinking and abstraction, an aspect of mathematics education not typically practiced which leads to students' aversion of mathematics [18]. The activities also embedded many problem-solving skills and met benchmarks such as: accurate plotting of coordinate points, graph shape (sound) prediction, data gathering, and communication of mathematical concepts.

In the first two sets of exercises participants were introduced to the mechanics of the software during rhythm & melody creation and reproduction exercises. Participants were invited to bring a popular music selection, or were presented with a choice of selections prepared by the author. Participants were then asked to listen for an individual melody or rhythm in the selection to be reproduced in the Sound Grid. Participants were also asked to enter tabular note data into the Sound Grid, make predictions about the next notes in compositions, and match sets of sound to sets of graphically and tabular presented data. In the third category of exercises participants were presented with sonifications of linear algebraic equations. The variances in representations (symbolic, tabular, graphic, and auditory) of resulting lines with changes to the leading coefficients of the equations were demonstrated. The concept of slope, y-intercept, and melodic contour was introduced at this time. Composition exercises comprised the final category of activities. Participants were encouraged to explore musical possibilities in their compositions and to use previously discovered composition techniques (linear series of notes, varying rhythms, contour). Students were also encouraged to discuss their musical ideas.

5. RESULTS & DISCUSSION

Figure 2 shows results of the MI test as combined intelligence categories(linguistic & logical-mathematical; musical & spatial; kinesthetic, interpersonal & intrapersonal). No participant chose all seven of the possible selections for an intelligence category; however, several made no selections in a category. The total number of selections per participant ranged from 11-30 (avg. = 18.9) of 49 possible selections.
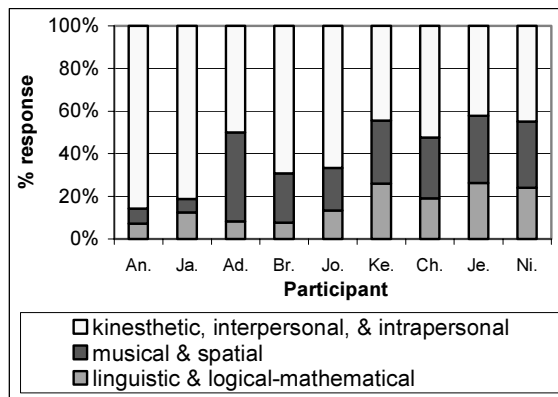

Figure 2. *Multiple intelligence test selections. Results shown as combination of intelligence categories.*

It is noteworthy that the participants' combined linguistic and logical-mathematical intelligence—presumably most valued in traditional instruction—generally scored lowest and may be indicative of alternative education needs. Four participants claimed learning disabilities effecting their "memory", and three demonstrated very poor English literacy or uncorrected sight problems that made reading difficult. Each participant came to the study lacking some basic mathematic skills (decimals, fractions, division, and/or multiplication) and all participants were poor in algebra. Midway through the study I removed decimal numbers and displayed rounded integers in the table for simplicity.

> *Ad. There will be some long words like, um, I can't think of a word right now, like, Mississippi. I can spell that cuz the way they taught me how to spell it: m-I-ss-I-ss-I-pp-I. [rhythmically]*

All participants enjoyed working with the curriculum and software as evidenced by use of the software outside of instruction sessions, requests to have copies of sound files created for personal use, spontaneous utterances, and interview responses when asked about the lessons. Participants also demonstrated engagement by bringing CD selections to sessions for the melody reproduction exercises (5 participants).

> *Ja. It was pretty fun stuff, like, how I did all the notes, and how I, like, you know, put together and stuff the beats and stuff.[sic]*

> *Ad. I'm sitting here having fun with the computer.*

The scheme of note sequence was a common question. Notions of visual order were guessed first. Most students entered notes from left to right, starting at the origin. These students rarely entered notes in the second or third quadrant. One student of obvious Asian descent began entering notes on the right of the grid and moved to the left. Other students freely used the entire grid space, and all participants eventually demonstrated this behavior. In the table window, the top to bottom presentation of note order caused no evident confusion.

Participants responded highly favorably to the interactive adjustments to sound character. "Wow", "That's tight", and "Cool", were common exclamations when shown how to adjust the grid scale, note durations, volume mapping in the x-coordinate, and instrument choices. In the case of instrument selection I found it difficult to direct participants away from sampling every possible instrument voice and toward creating new compositions. Participants commonly searched for specific instruments, were unfamiliar with many instruments, and upset when an instrument did not sound as expected. The "Draw" mode was intriguing when first used, but participants (and I too) quickly tired of the poor control over exact note placement.

Exercises that went smoothly involved composition. Some participants preferred activities focused on rhythms while others preferred activities focused on melodies. Making "hip beats" was the reason several students participated. The inability to multi-track compositions disturbed several participants who were interested in layering several beats and instruments. The playback "Loop" option was essential.

Participants found rhythm and melody reproduction activities challenging. All were interested in the idea of the activity, particularly to take singing voices off a song. In practice, the rigor of the activity quickly became apparent and some participants lost motivation. The complexity of the music sample is an important determinant in this activity. Several participants seemed confused by the concept of multiple tracks, referring to the entire non-vocal "background" as a whole. Naturally, participants wanted to reproduce interesting, complex, multi-track instrumentals (only one participant was interested in a vocal melody), so identifying basic, reproducible rhythms within a musical sample was my task as the educator. The discourse in this situation involved critical listening to the sample for individual voices and rhythm recognition, often by tapping on the table or vocalizing the track of interest. Indeed, participants found it extremely difficult to communicate which track they were interested in otherwise. To enter the track in the Sound Grid a worthwhile starting point was counting the number of individual notes in a rhythm, followed by making the note durations accurate.

When reproducing CD samples was too difficult we worked on reproducing simple rhythms tapped on the table. Even this proved very difficult for two participants who exhibited inability to consistently tap out a steady rhythm. One participant needed a whole session to reproduce a 4-8-4-8 rhythm. It is not surprising that these two participants scored lowest of all participants in musical intelligence. On the other hand, the concept of numerical note durations was natural for other participants, especially those with a background in music performance. It took little or no explanation of the software interface for them to accurately reproduce rhythms.

Many students picked up new terminology for graphic features (e.g. axis, y-intercept, slope) and had personal ways of expressing the meaning of the terms that were generally accurate when questioned about the details. Participants were also comfortable describing notes in terms of numerical attributes. Often, when I asked how one could identify a melody in the Sound Grid from several tabular choices I was told a series of numbers.

> *R.U.   Between this line and the line from the last one, what's the difference?*
> *Ja. Oh. This one go right there, and this one curving in.*
> *R.U.   This one's curving? They're both straight lines, OK? See?*
> *Ja. What I'm trying to say is, the last line was something like this, right? And now, now this line's like this now. [pointing steeper]*

> *[rhythm identification activity]*
> *R.U.   How do you know that [is the rhythm]?*
> *Jo. It goes double, single, double, single.*

Participants became very proficient at using the table to edit sound data. All participants were able to identify correct data sets from the graphic representation and to accurately create graphic representations in the grid given a table of note durations or pitch values. Initially this task confused participants since, apart from the table headings, the numerical data for pitch and duration looked identical. A second and sometimes third set of verbal instruction finally cleared up any misunderstanding about the nature of the task. This activity encouraged later participant creativity by easing note identification and note placement. Participants commonly listened to compositions, identified notes of interest (either visually when redrawn in the grid during playback, audibly, or by inference from known notes found as previous) and modified tabular values of rhythm and pitch. Participants made associations between number and note, and referenced numbers relatively as "higher" and "lower" which led to confusion when the sound quality under consideration was not pitch.

> *Ad. "That one. Is this the right one here? Nope because there's two 16's instead of one and it begins with an 8. And these are the only two that begin with an 8, oh no, that doesn't begin with, that begins with a 4. OK. 4, 8, 4, 4, right there."*

> *R.U.   For instance do you know what this one might sound like?*

*Ni. Kind of. It kind of tells me that it's
going to be back and forth a little bit. It
goes from high to low to very high to back
down lower.*
R.U.   What do you mean high and low?
Ni. Uh, no. Longer and shorter.

Exercises using the algebraic equations to compose were
not as well received. Creating many lines, listening to results,
emphasizing the compositional utility of lines, searching for
patterns in the graphs of different lines, and entering a
coefficient into a text box to create a line was much easier than
using equations to calculate notes one at a time. However, one
participant initially believed that slope was determined by the
pair of numbers entered in the slope equation window when in
fact one of the numbers was the slope and one was the number
of points to draw along the line. When participants were asked
to determine the slope of a line from its points using the
equation $\Delta y/\Delta x = slope$, or to determine the y value of a point
given a slope (m) and an x value using the equation $y = m*x$, I
was faced with blank stares and yawns.

Ch. All this a, b, c, stuff confuses me.

*R.U. Do you remember, we were talking
about slope?*
*An. Slope? Yeah. Like x equals y and then
something. Like something something and
then the slope. The slope is the number.
[sic]*

The activities had little effect on the participants' math
achievement as measured by standardized test results. Figure 3
shows the change from pre to post-tests results. These results
are not surprising given the lack of familiarity with algebraic
manipulations that comprised much of the test. The increase in
the number of questions answered shows students' comfort with
the subject improved. No student answered every question on
the test (pre or post) demonstrating that they must have felt
some degree of knowledge with the test question to answer it, as
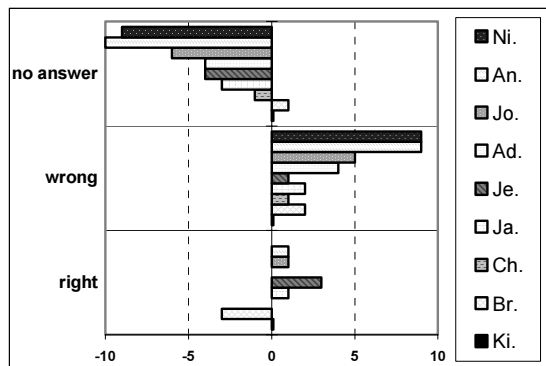the test instructions stated.

Figure 3. *Change in results from standardized pre- to post-test.*

Winsløw [19] outlines a theory of linguistic development
necessary to form traditional mathematics knowledge. His
inventory describes basic semantic and syntactic revisions of
natural languages that take place for mathematical knowledge
acquisition and communication. As previously shown, the
participants in this study had relatively poor logical-linguistic
memory and abilities. Therefore, it makes sense that they

would show little improvement in evaluations that emphasize
logical-linguistic flexibility, as the sample test questions
exemplify, with no extra training in symbolic manipulations.

*An. I keep, I keep forgetting this is, um, I
keep thinking this is times tables for some
reason…it's kind of like this, like, if you
got, like, the parenthesis thing.*

Figure 4 breaks down test results by two question types:
those that involved equations (example questions A & B above)
and those that did not involve equations (example questions C
& D above). Results are given as percentages of responses in
each question type for all participant answers. This analysis
shows that participants answered more questions that did not
involve equations, and answered more of the non-equation
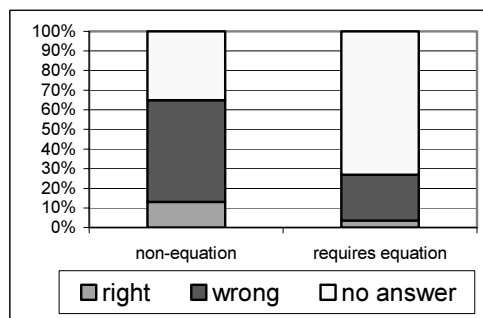questions correctly.

Figure 4. *Combined results on standardized post-test by
question type as percent within question type.*

The value of the software for the students came in music
composition. Honestly, algebra was of little use to the students
for composing. The concept of melodic contour in linear
ascending and descending series of notes created by algebraic
equations was too complex and sterile. The Sound Grid
allowed participants to freely enter notes, sometimes in lines,
sometimes not. In order to improve algebraic ability, exercises
that involve symbolic manipulations, or that mimic procedural
operations, should be practiced. The participants did
demonstrate that they could follow procedural instructions and
infer meaning from symbolic representations by reproducing
melodies and rhythms, describing auditory patterns, and using
visual space to create and edit notes.

If traditionally measured skill in algebra is the learning
objective, buttons or sliders that procedurally adjust note or
composition character may be a starting point for further
refinements to the software. Practicing procedural tasks would
shift the utility of the software and the understandings users
take away from the experience [20]. Operative adjustments to
sound quality (add 3 to the pitch value) rather than discrete
adjustments (change the pitch to 17) could also highlight
interrelationships between sound qualities and alter users'
lexicon and grammar of composition activity to more closely
match the language of mathematics [21].

Alternatively, an assessment could be designed to
accommodate the preferred intelligence of the student. The
musically able participants demonstrated exceptional ability to
memorize and create patterns of sound. The academic culture
does not accommodate this intelligence in assessment and thus
many participants had stigmas about their academic abilities.
What would musical mathematics assessments sound like? If
measures of musical operations mirroring those of the symbolic

manipulations in algebra were developed the assessments would probably require technological assistance to administer. It is feasible to imagine the development of evaluations of auditory files rigorous enough to satisfy traditional mathematicians who may otherwise not recognize the mathematics in aural mathematics education [9].

> *Ni. How can I remember that beat?*
> *R.U.   You   got   to   remember   it*
> *unfortunately.*
> *Ni. Can I copy that down?*
>
> *An. I'm still trying. I'm just trying, trying to picture it in my mind, trying to remember it.*
>
> *Ch. We tried to make that [taps on table]. I remember good.*
> *R.U.   Oh. You still got the beat?*
> *Ch. I was actually playing it on the keyboard at Radio Shack.*

6.   CONCLUSION

Sonification-enhanced curriculum provided an engaging experience for non-traditional learners. Though inclusion of sonifications attracted students to mathematics, they had difficulty transferring understanding of auditory and graphic representations to abstract mathematics principals and procedures as traditionally represented. This difficulty was particularly evident as poor results on a standardized test and participant comments regarding symbolic mathematic procedures. Refinement of the sonification software and curriculum to include more procedural manipulation and alternative assessment techniques are suggested to raise student mathematics achievement.

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