

# Sound Synthesis and Composition Applying Time Scaling to Observing Chaotic Systems

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## Abstract

This chapter presents a working model for bringing computational models to cognitive process by designing auditory signals for the models. *Auditory structure* is defined as an observed structure which is meaningful with respect to data from the computational models. Three different applications were designed and used for generating classes of sound examples from an iterative model and a continuous model of chaos. The concept of *explicit* and *implicit time* are introduced as concrete timescaling attributes to the computational models and synthesis engines. In addition, examples applying *tempo*, *rhythm*, and *conductor* are presented in order to generate cases to make steps towards the general principles for constructing *temporal grammar*. These three applications were evolved as model-specific, meaning their designs are based upon the different kinds of complexity and problems posed by computational models having to be addressed in the synthesis design process. For descriptions of auditory signals working vocabularies such as *auditory crossfade*, *timbre chord*, and *timbre rhythm* are introduced in the context of experimental reports involving specific sound examples.

## 1 Introduction

Where human intellect acts, there are structures. It has been often said there are structures everywhere in nature. In this saying the presence of an observer is transparent with an assumption that we imply an observer. The assumption has transcended the convenient omission of an observer and the omission has been stabilized into our grammar. Then the grammar governs our thoughts and the observer is forgotten. And we still say, "there are structures everywhere in nature" in which the forgotten observer no longer exists.

Structure found in nature is the first evidence that we project our observation into nature. Poking above the ocean in red, yellow, and blue, at a Hawaiian beach the busy movements of snorkeling tubes of underwater crowds tell us, we want to exist and we want to see. Our observers want to exist and be rebuilt into our grammar. Through projects such as sonification we create *an art of observation* bypassing the politics of the adjectives "scientific" or "artistic" attached to the noun *observation*.

### 1.1 Sociological Construct for a Working Model

Bringing computational models to a sound synthesis environment is an interdisciplinary project. For generating nontrivial descriptions of this interdisciplinary project I would like to bring the concept of *con-scientia* (con-science), "together-knowledge." This terminology was introduced

## Bringing numerical data into cognition

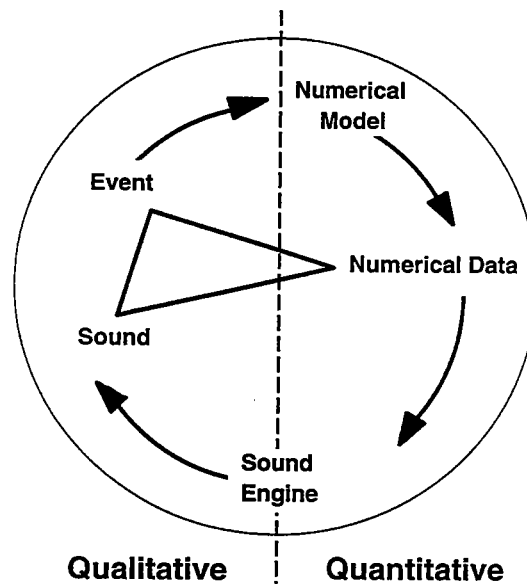


Figure 1: Observation cycle from a computational model to sound.

by Heinz Von Foerster to account for the knowledge that can be shared [1]. Reality can be witnessed—rather than taken for granted—based upon this knowledge, and is supported by the presence of a second observer. Without the second observer one takes a risk of *solipsism* [2]. The project of sonification may offer an interesting case when researchers take each other's field as a second observer. The binding motivation for multiple disciplines in this project is to construct an observatory platform to enhance con-science. And the con-science is nourished by multiple descriptions offered by multiple ways of looking and describing learned from multiple disciplines, thus expanding the margins of self-reflections among disciplines.

### 1.2 Bringing Data from Computational Models to Cognitive Processes

Listening is observing. By listening to the acoustic signals which report the internal state of a system we bring the system's internal process into our cognition. This way of making observation is making qualitative observation. Figure 1 is a summary of how qualitative observation takes place involving computational models [3]. A computational model may function as a vehicle to control sound rendering systems [4].

The term *qualitative observation* is introduced as an alternative to “qualitative properties of scientific data.” The latter term seems to be commonly accepted in the scientific community. However, do scientific data and computational models really have qualitative properties? Numerical data has *intrinsic properties*, and we may gain knowledge of intrinsic properties by performing analysis or by rendering the data in a qualitative form. “Knowledge” is not the same as “quality.” We do not know qualitative properties until we bring the intrinsic properties of numerical data into our perception. Until a listening experience is constructed we cannot say how intrinsic properties “should” sound. Sound synthesis and listening with respect to a computational model offer a platform for experimental observation. To better interpret scientific data we can gain an understanding of the rendering methods underlying scientific observations.

### 1.3 Qualitative Observation and the Role of Composers

Computational models could be based upon the simulation of events which take place in the real world, or upon physically based models, or upon composed algorithms. They generate numerical data that is to be rendered through a sound engine. With the sound engine the data is transformed into sound for an observer to make qualitative observations. Notice the sound engine bridges the quantitative to qualitative platform for converting the mode of observations in figure 1. There is a gap where an observer's active cognitive process takes place, from "sound" to "event." In this gap an observer makes hypotheses, retroactive corrections to previous hypotheses, adding irreducible qualities to the understanding of the event under study while pondering the correlation between sounds and the events. This is also the gap where an observer makes a self-reflection on her own cognitive process prior to her next actions, such as coming up with a new hypothesis, and constructing descriptive or explanatory principles.

The role of a composer is to understand the complexity of the data for designing acoustic characteristics to reflect or represent the data without reducing the complexity of the data. Then she examines the sound rendering system to determine whether the system is capable to serve for producing the desired variety of acoustic signal characteristics. When it is necessary the sound engine may be reconfigured or new functionalities may be added to it.

Synthesized sounds may be controlled from computational models. The sound designer determines their relation based upon criteria contributed by scientific knowledge of computational models and human perception. Thus we conceive scientific analysis as valuable criteria and guideline, not as an absolute prescription how we ought to design and perceive sounds. Section 2 discusses the role of listening in detecting structures in sounds. Section 3 discusses two computational models of chaos and timescale methods for attaching them to a sound rendering engine. Section 4 presents experimental reports of specific techniques for applying sound synthesis to observe chaotic signals. Section 5 presents an interface for multidimensional control. It is important to remember the sounds we perceive come from a rendering engine, not straight from a computational model. Even when the computational data are used as sound samples, they need to be brought into scale for human perception. Timescale techniques are always present in synthesized sounds.

## 2 Auditory Structure and Signals

The variety of usages of the term *signal* include that of engineering theory [5], information theory [6, 7, 8], and semiotics [9]. In this chapter the concept of the term, *acoustic signal* embraces those of *tone* and *noise* attributed to sound by Helmholtz [10]. He differentiates noise from musical tones in the following:

The nature of the difference between musical tones and noises, can generally be determined by attentive aural observation without artificial assistance, . . . , the irregularly alternating sensation of the ear in the case of noises leads us to conclude that for these the vibration of the air must also change irregularly. For musical tones on the other hand we anticipate a regular motion of the air, continuing uniformly. . .

The term *sound*, accounts for both acoustic signals and auditory signals. These terms are often interchangeable with an emphasis in usage of *auditory signal* in respect to perception. As a similar case, composers make a subtle distinction between the verbs *hear* and *listen*, the latter describing an attentive listening whereas the former accounting for broad sensory responses to acoustic signals.

Description	Meaning when applied to these subjects:		
	mathematics	type of simulation	class of auditory signals
“discrete”	summation	digital simulation discontinuities	note events and silences;
“continuous”	integration	analog hardware; physical air pressure	no silences; smooth transitions between states

Table 1:

## 2.1 Auditory Structures as Emergent Properties

Auditory structures emerge from the perception of temporal changes in auditory signals. Since auditory signal implies perceived signal, the formation of auditory structure is largely contributed by listener’s perception and ability to extract some structure from auditory signals. When we consider external factors to influence listener’s cognitive process such as sound rendering systems and computational models of our interest, auditory structure may be considered as an emergent property of interacting complexes in the observation cycle, from listener to acoustic signals to computational models. The term *emergent properties* is often used in the field of neural networks to refer to “properties that are not programmed but emerge somehow from the complex of neuron-neuron interactions” [11]. With an understanding of the interacting complexes in the observation cycle in figure 1 one possible working thesis for the project of sonification can be made: *we do not know how period 5 sounds until we listen.*

## 2.2 Continuity and Perception of Continuity

Descriptions of timeseries data from computational models or simulations often emphasize their properties as *discrete or continuous* signals.<sup>1</sup> These two terms have different meanings for different classes of signal, including physical and analog signals, digital signals; numerical representation of signals, and perceived signals (see Table 1). When we refer to sounds as discrete or continuous we want to consider the following: (1) representation systems and sampling methods, (2) structural organization and perception.

### 2.2.1 Representation and Sampling

According to what medium we use in order to store, process, analyze, or produce signals, signals have to be *re-presented* when they are sent from one medium to another. In sound synthesis involving digital technology signals are broadly classified into two kinds, digital and analog. Acoustic signals can be captured, stored, and manipulated. Transducers such as microphones and loudspeakers convert between variations in air pressure and variations in electrical voltage. Analog electronic signals are composed of continuous waveforms, analog to the vibrations of air pressure from a sound. When we use a computer as a medium to store acoustic signals they have to be transformed into digital signals. This process requires several stages and is well illustrated in Dodge and Jerse [15]. Values from continuous waveforms of analog signals are extracted at regular time intervals and converted from voltages into numbers called *samples* so a computer can store and process them. Sound is represented in a computer by arrays of samples, defined as integer or floating point numbers. This representation of an acoustic signal in a computer is called a digital representation of the signal, and arrays of samples are digital signals.

<sup>1</sup>The discussions of continuity and perception of continuity may concern in broad sense the issues of auditory segregation and streaming [12, 13, 14].

For more specific classification of signals on an operational basis, Garnett classifies musical signals into three kinds; acoustic musical signals, analytic musical signals, and parametric musical signals [16]. In this chapter we concern with (1) signals from computational models to which we frequently refer as *data* and (2) acoustic signals from a synthesis engine which renders signals from parameterized functions, and (3) establishing relationships between these two classes of signal. Signals from computational models do not yet belong to any musical signals in Garnett's classification. Those signals can be parameterized internal to or external from the computational model before they are sent to a synthesis engine to be rendered into acoustic signals. When the parameterization is internal to a computational model, the parameterization has to be compatible to the synthesis parameters so that the computational model can send parametric musical signals to control a synthesis engine. When the parameterization is external to computational models, an intermediate step has to be implemented in order to turn the signals from computational data into parametric musical signals.

### 2.2.2 Structural Organization and Perception

Iterative and continuous functions are different in their mathematical description. However, classifying this difference in terms of auditory perception is rather meaningless. Both discrete and continuous signals can produce both discrete and continuous perceptual characteristics. Eventually all auditory signals come from physical and continuous variations in air pressure. Perceptions of discrete sounds come from interruptions and discontinuities in the flow of signal information, including rests, pauses, silences, and *bifurcations*.<sup>2</sup> However, even a bifurcation scenario in a chaotic system such as Chua's circuit can be perceived as a continuous event when the onset of state changes is sufficiently regular and the time interval between changes is sufficiently short (see experimental report in section 3.3). Further, the perception of continuity and discontinuity can be applied to higher organization of musical events in terms of formal structure. For compositional projects rests, pauses, and silences are articulation tools for constructing *temporal grammar*. We define temporal grammar as the system of rules for generating sounds and silences. The system of rules we concern here may be derived from computational models.

## 3 Sound Synthesis Applied to Observing Chaotic Systems

This section describes rudimentary principles of computational models of our interest, to prepare readers for the discussion of specific synthesis techniques applied to these models. The more detailed pedagogical information of these models can be found, for the logistic map in Garnett [18] and Abraham and Shaw [19], and for Chua's circuit in Chua et al. [20], and Chua [21]. Each model is connected to a synthesis engine. In each case two issues will be examined: the *implicit* or *explicit* time characteristics of computational models, and the *rendering ratio* between computational model and the synthesis engine.

Implicit and explicit time characteristics refer to the perceptual consequences of parameters in a computational model. It will be important here to distinguish between sequential and temporal attributes. In scientific practice a sequence of values from a computational model is referred to as a "timeseries" using an abstract notion of time that does not include a specification of duration. A timeseries without duration offers sequential attributes. We refer to this abstract notion of time

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<sup>2</sup>*Bifurcation* is understood in science and engineering as a natural property of instability where rapid changes in states of a system occur given some control value or force. This abrupt change causes a major transformation in the phase portrait of the system dynamics; the term bifurcation refers to the major transformation in the phase portrait [17]. Examples of bifurcations in signals from Chua's circuit are discussed in section 4.3.

as implicit time. Implicit time parameters influence the sequential output of the model but their influence does not include a duration specification. Explicit time parameters provide explicit duration. A timeseries with specific durations offers temporal attributes as well as sequential attributes.

The rendering ratio between a computational model and a synthesis engine is specified in terms of *one to many*, *many to many*, and *one to one*. A one to many ratio indicates for every data sample a series of sound samples is produced; this approach is applied to render an iterative sequence from the logistic equation. Many to many indicates many data samples are used as a group to influence a series of sound samples. This approach is applied to render a statistical evaluation of the logistic equation. One to one indicates one data sample is used to control one sound sample; this technique is applied to render signals from a simulation of the Chua's circuit.

### 3.1 Computational Models of Chaos

Nonlinear dynamical systems provide a class of numerical models producing signals evolving over time in often unpredictable ways, due to nonlinearities in the systems. Physicists have borrowed the term *chaos* to refer to the deterministic yet unpredictable behavior exhibited by these models. Signals from these systems exhibit characteristics ranging from simple periodic patterns to complex, noisy, and seemingly random behaviors. These signals can be characterized by a *mixing mechanism*, referred to as *stretching and folding*: a unit interval defined by nearby initial points is stretched apart repeatedly by exponential growth until it reaches the ergodic limit of the system then the two initial points are folded back together. While the stretching operation tends to pull apart nearby orbits, the folding operation brings them back together within a bounded region of phase space. In addition to the mixing mechanism, stretching and folding, a chaotic system displays an asymptotic motion that is not an equilibrium point, periodic, or quasi-periodic, and this motion is often called chaotic motion. Further this chaotic motion is bounded to an asymptotic solution that possesses sensitive dependence to initial conditions [18, 19]. *Attractors* are characteristic signal trajectories on or near convergent regions of the phase space of these systems.

Chaotic systems can be described either in discrete iterative maps or in continuous trajectories. In mathematical terms, the iterative maps may be expressed in finite difference equations, the continuous trajectories may be expressed as a set of ordinary differential equations (ODE's). To study application of timescaling techniques we will study aspects of an iterative system, the logistic map, and a simulation of a continuous system, the Chua's circuit.

### 3.2 The Logistic Map

Because of the availability of its extensive study, the logistic map provides a good starting point for producing sounds of iterative systems. The logistic map belongs to a class of nonlinear equations that derive successive values from preceding values [Eq. (1)]

$$x_{n+1} = f_r(x_n) \quad (1)$$

with the nonlinear function [Eq. (2)]

$$x_{n+1} = f_r(x_n) = rx_n(1 - x_n) \quad (2)$$

where  $r$  determines the rate at which increases or decreases take place. The logistic equation is represented as a return map by plotting  $x_n$  on the abscissa and  $x_{n+1}$  on the ordinate axis (see Figure 2). A parabola represents all the possible values for  $x_{n+1}$  given  $0 \leq x_n \leq 1$ . The diagonal line reflects  $x_{n+1}$  onto the  $x_n$  axis for generating the next member of the series.

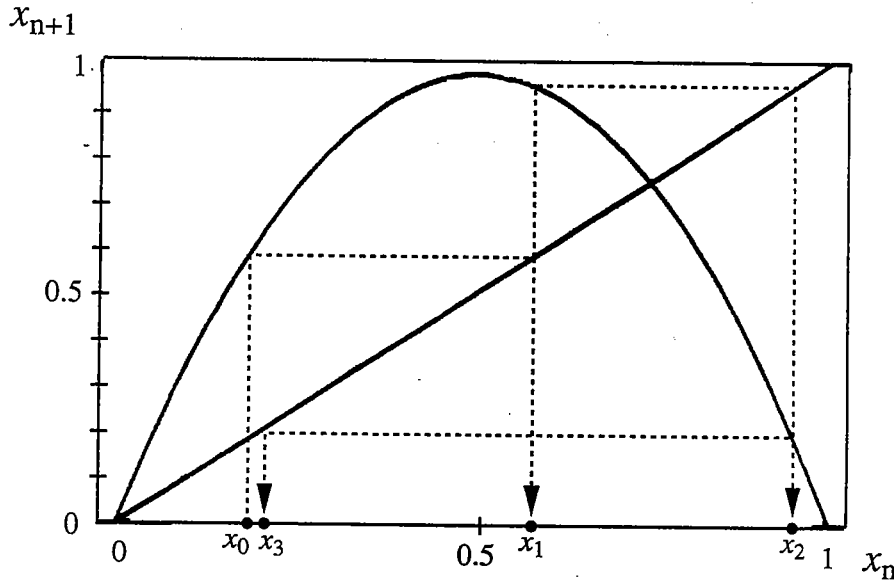


Figure 2: Map of the logistic equation, showing three iterations plotted from point  $x_0$ .

### 3.2.1 Signals From the Logistic Map

Signals from the logistic map are iterated sequences and may be described in terms of orbits. An orbit may be simple, including relatively few values, or complex, containing many values. In the first case repetition may be easily noticed; in the second case repetition may be difficult to notice. Orbits are referred to as *attractors* in chaotic systems. In the logistic map for  $1 \leq r \leq 4$  the range  $0 \leq x_n \leq 1$  may be called the “basin of attraction” since all sequences with values of  $x_0$  chosen within this range will eventually converge onto an attractor. The  $r$  value determines the attractor. The initial value of  $x$  with respect to  $r$  determines the complexity of the signal as it converges onto the attractor. To identify attractors an initial number of iterates (often as many as 5,000 or more) is discarded before studying the sequence. Figure 3(a) and 3(b) show a periodic and a chaotic attractor from the logistic map for two different values of  $r$ , obtained after discarding 500 iterates. Figure 3(c) shows convergence onto a period-five attractor.

### 3.2.2 Implicit Time in the Logistic Map

Iterative functions such as the logistic map do not include an explicit time parameter. In the logistic equation, the historical dependence of each value of  $x_n$  upon  $x_{n-1}$  implies the significance of sequence. The sequence implies time to the extent it specifies the order of events, not the duration.

In order to bring the timeseries data of this model into auditory signals a time interval,  $T_i$  is specified in explicit time duration. This interval determines how much real time passes in the synthesis engine between the application of successive values from the logistic map. Given a constant time unit  $T_i$  we select a rendering ratio to specify the number of sound samples generated with respect to a single iteration of the logistic equation. The one to many ratio indicates the logistic map provides a scalar value rather than detailed acoustic signal characteristics. Each logistic value is applied as a scalar to a series of samples; the series of samples obtain detailed acoustic signal characteristics from synthesis functions in the rendering engine. Section 4.1 presents experimental results applying a one to many ratio to the logistic equation; section 4.2 presents results applying a many to many ratio to the logistic equation.

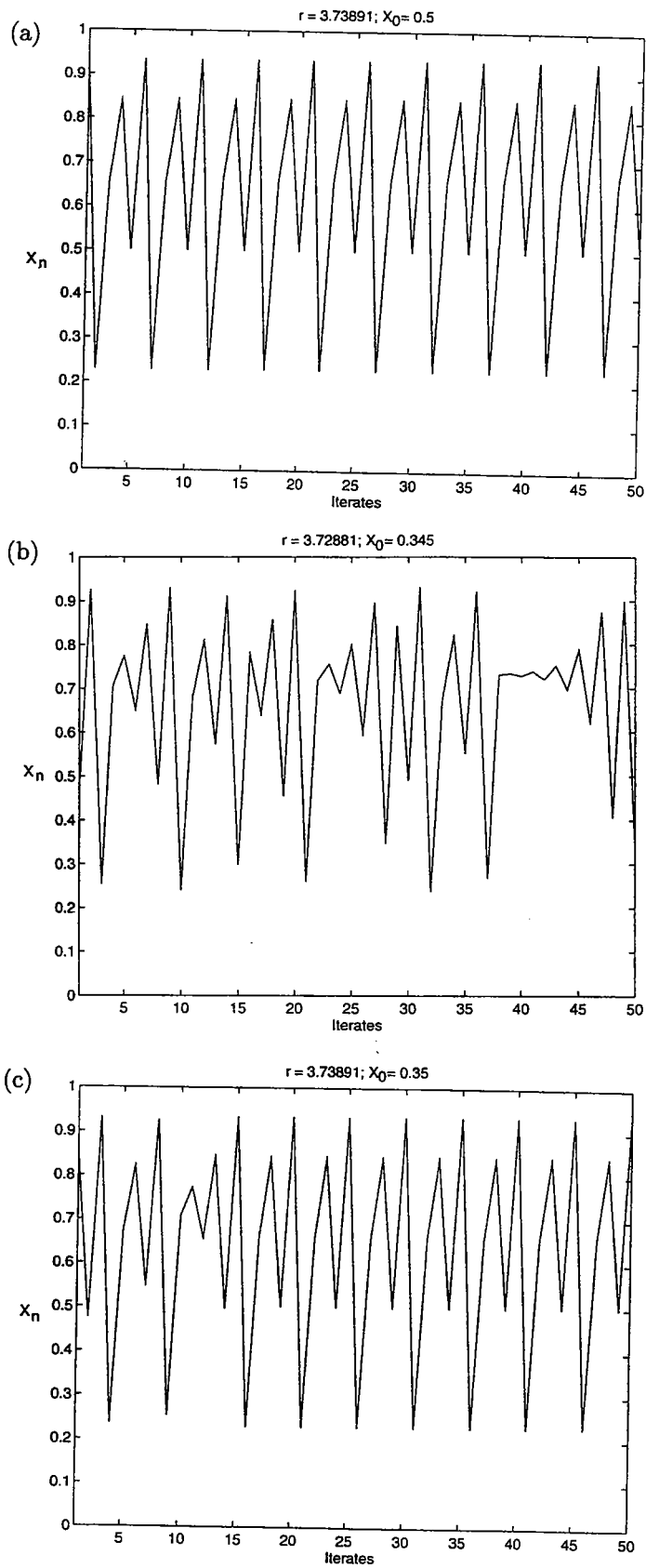


Figure 3: Signals from the logistic map. Each signal is 50 iterations in length. (a) depicts a period-5 attractor after 500 iterates were discarded. (b) depicts a chaotic attractor after 500 iterates were discarded. (c) depicts an initial 50 iterates converging onto a period 5 attractor.



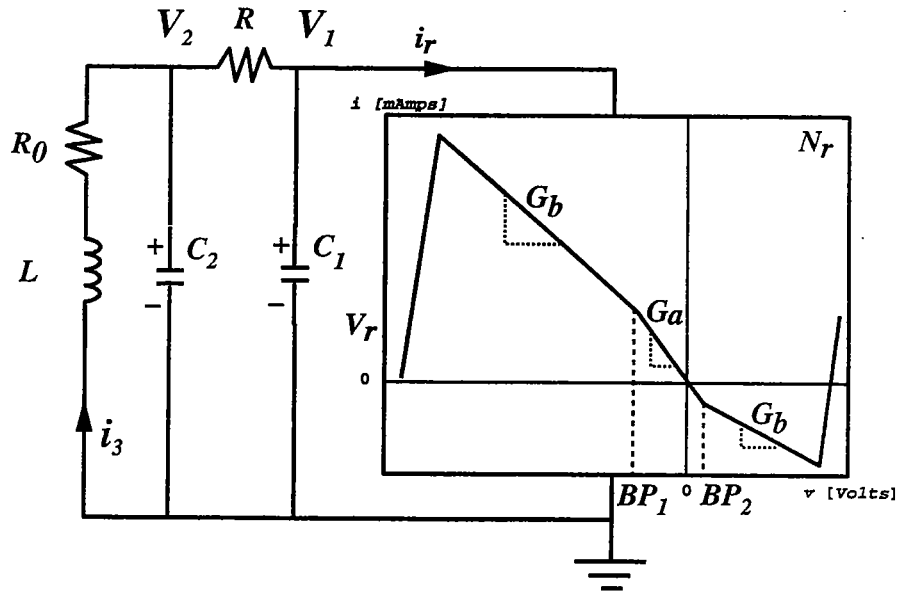


Figure 4: A diagram of the unfolded Chua's circuit, also showing the  $v - i$  characteristic of the nonlinear resistor.

### 3.3 The Chua's Circuit

The Chua's circuit is an autonomous dynamical system that generates continuous signals. It is also chaotic fulfilling the working definition for chaotic systems [21, 22]. The Chua's circuit is an electronic circuit which is designed to produce chaotic behavior from an analog system. It is made up of the minimum number of components that a circuit requires in order to demonstrate chaotic behavior [23]:

1. one locally active resistor
2. three energy storage elements
3. a nonlinear element

The basic elements of the Chua's circuit include four linear circuit elements and a nonlinear resistor  $N_r$  which is called the *Chua's diode* [23]. The four linear elements consist of an inductor  $L$ , two capacitors  $C_1$  and  $C_2$ , and a resistor  $R$ ; a linear resistor  $R_0$  is added in series to  $L$  (see Figure 4) in order to explicitly model the resistance characteristics of  $L$ . Chua's diode has a piecewise-linear driving point characteristic including a negative resistance that allows energy to be introduced into the system. The piecewise-linearity in  $N_r$  includes three segments with negative slopes; the negative incline of the slopes describes the amount of energy introduced into the circuit. Increasing the negative incline of the slope increases the amount of energy introduced.

The system dynamics of the physical model of Chua's circuit can be described by a system of three ordinary differential equations referred to as the *global unfolding* of Chua's circuit [21]:

$$\begin{aligned}
 \frac{dv_1}{dt} &= \frac{1}{C_1} [g(v_2 - v_1) - f(v_1)] \\
 \frac{dv_2}{dt} &= \frac{1}{C_2} [G(v_1 - v_2) + i_3] \\
 \frac{di_3}{dt} &= \frac{1}{L} (v_2 + R_0 i_3)
 \end{aligned} \tag{3}$$

where

$$G = \frac{1}{R}.$$

These equations account for three parallel signal paths, the voltages crossing two capacitors ( $C_1, C_2$ ), and the current passing through the inductor ( $L$ ). They are denoted by  $v_1, v_2$ , and  $i_3$  in Eqs. (3) and (4). These make a three-dimensional signal, which is required for a system to exhibit chaotic behavior. Another requirement is a nonlinearity, provided by the function  $f(v_1)$ , defined by:

$$f(v_1) = Gbv_1 + \frac{1}{2}(G_a - G_b)\{|v_1 + E| - |v_1 - E|\} \quad (4)$$

*Unfolding* refers to the theory in mathematics of adding parameters to dynamical equations—in this case  $R_0$  added in the third differential equation—in order to generalize their dynamical behavior [21]. In the case of Chua's circuit these equations are defined over the entire state space of the circuit signal, and referred to as global. This circuit has been referred to as the *Chua's oscillator* [24]. It is canonical<sup>3</sup> with a large class of three-piecewise nonlinear electronic circuits.

### 3.3.1 Signals from Chua's Circuit

One can extract varieties of signals from Chua's circuit ranging from simple periodic patterns to noise-like chaotic signals. Whereas the logistic map provides a single control parameter  $r$ , bifurcations may be created in the signal from the Chua's circuit by varying as many as six specified circuit parameters [25]. Influencing the state of the system by varying circuit parameter values is referred to as *parameter variation technique* [26]. Figure 5 demonstrates signals obtained by applying parameter variation technique. These figures show the short-time fourier transform (STFT) and waveform representation of each signal. To change the state of the system from chaotic to periodic, two parameters,  $R$  and  $BP_2$  are varied with the other parameters fixed.

### 3.3.2 Explicit Time in the Chua Circuit

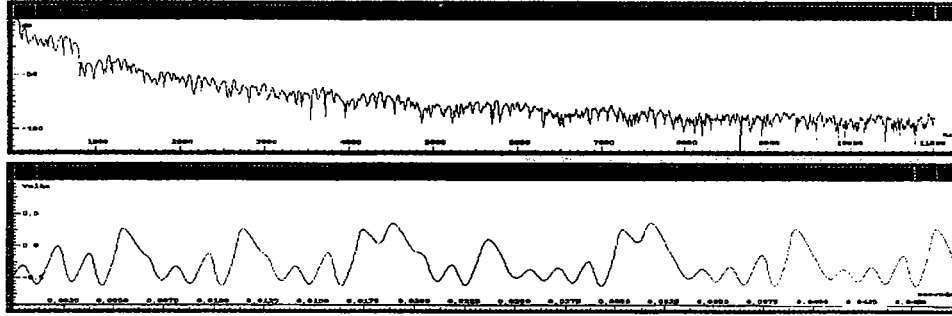
ODE's are modeled in the computer as Ordinary Difference Equations, producing discretized signals with potentially smooth trajectories in phase space. For integration a discrete time step  $\Delta\tau$  is assigned to  $dt$  [Eq. (3)].  $\Delta t$  determines the time interval at which successive samples are drawn from the continuous trajectory described by the ODE's. In order to use  $\Delta\tau$  as an auditory timescale parameter, a *one to one* rendering ratio is selected. A one to one ratio indicates one sound sample is generated for every data sample. To apply timescaling we control the relationship between the time interval  $\Delta\tau$  and the time interval of the sample rate (SR) of the D/A conversion. Simply put, samples are computed with respect to  $\Delta\tau$  and displayed with respect to SR. Frequencies in the output signal can be raised or lowered by changing the ratio of  $\Delta\tau$  to SR. We wish to keep SR constant in order to maintain the predictable frequency resolution of the output signal, therefore we specify the time interval  $\Delta\tau$  in terms of SR. For a given sample rate,  $\Delta\tau = C/SR$ , where  $C$  is a constant determining the ratio between  $\Delta\tau$  and SR.

For an example let us take a periodic signal producing a well-tuned fundamental frequency. Figure 6 shows a period-8 attractor from the Chua's circuit simulation. We cannot tell the fundamental frequency of the period until we know two things: the sample rate at which it is reproduced and the number of samples contained in one iteration of the period. For  $C = 660.0$ ,  $\Delta\tau = 20.625$  ms, producing a period-8 cycle every 340 samples. For SR = 32 kHz, this period-8 cycle composed of 340 samples results in a frequency of 94.118 Hz. Increasing  $C$  the same

<sup>3</sup> *Canonical* means that the circuit trajectories in vector space are topologically conjugate ("qualitatively equivalent") with a large class of vector fields generated by 3-D circuits containing a piecewise-linear nonlinearity [21].

### chaotic double scroll attractor

$R$  1398  $\Omega$ ,  $R_0$  11.2  $\Omega$ ,  $c_1$  .47 nF,  $c_2$  10 nF,  $L$  57.4 mH,  $M_0$  -0.5  $\Omega$ ,  $BP_1$  -1.0 V,  $BP_2$  1.0 V



### period one stable limit cycle

$R$  1688  $\Omega$ ,  $R_0$  11.2  $\Omega$ ,  $c_1$  .47 nF,  $c_2$  10 nF,  $L$  57.4 mH,  $M_0$  -0.5  $\Omega$ ,  $BP_1$  -1.0 V,  $BP_2$  0.15899 V

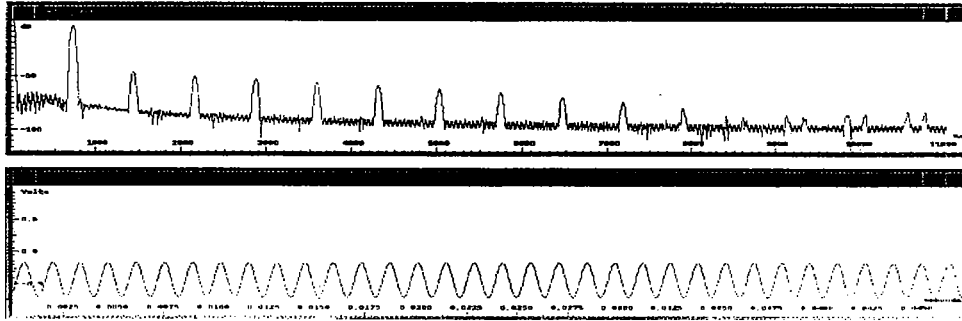


Figure 5: A Fourier spectrum and waveform from two Chua's circuit signals, a period-one limit cycle and a chaotic signal. The Chua's circuit signal was varied between these two signals by changing the parameters  $R$  and  $BP_2$ .

trajectory is rendered with fewer samples, which signal converted at the same SR is perceived at a higher frequency. Similarly as  $C$  decreases, the output frequency decreases. This timescaling method preserves a specified sample rate while optimizing frequency range.

## 4 Time Scaling and Control: Tempo, Rhythm, Conductor

This section provides experimental reports of timescaling applied in sound synthesis. The capability of a given numerical pattern to create different perceptual features at different timescales is a well-known sound synthesis principle articulated by several pioneers of electronic music (see for example Eimert [41], Koenig [42], and Stockhausen [43]). The following reports describe the results in auditory signals with respect to signals from computational models described in section III. In order to account for the variety of timescaling and control methods applied to computational models, the terms tempo, rhythm, and conductor are introduced.

Tempo is one of the most elementary specifications for timescaling in music composition. Tempo is used in reference to beat and meter, and it provides speed for pulse which is the most elementary countable unit, often constant. *Beat* is an intermediate timescaling unit for grouping pulses, *meter* is the next higher unit to specify the organization of beats in groups. The perceptual effect of tempo is often described in terms of "fast" or "slow". We will use the notation,  $T_i$  for timescaling for tempo and in section 4.1 we will study how  $T_i$  effects the observation of a computational model, in this case the logistic map.

*Rhythm* is an onset pattern created by modulating constant pulses of which durations are specified by  $T_i$ . The modulation comes from a computational model, in this case the mechanism for a *listening memory* (see 4.2) involving statistical evaluation of outputs of the logistic map. In section 4.2, the notion of timbre chord accounts for the degree of perceived complexity of tones

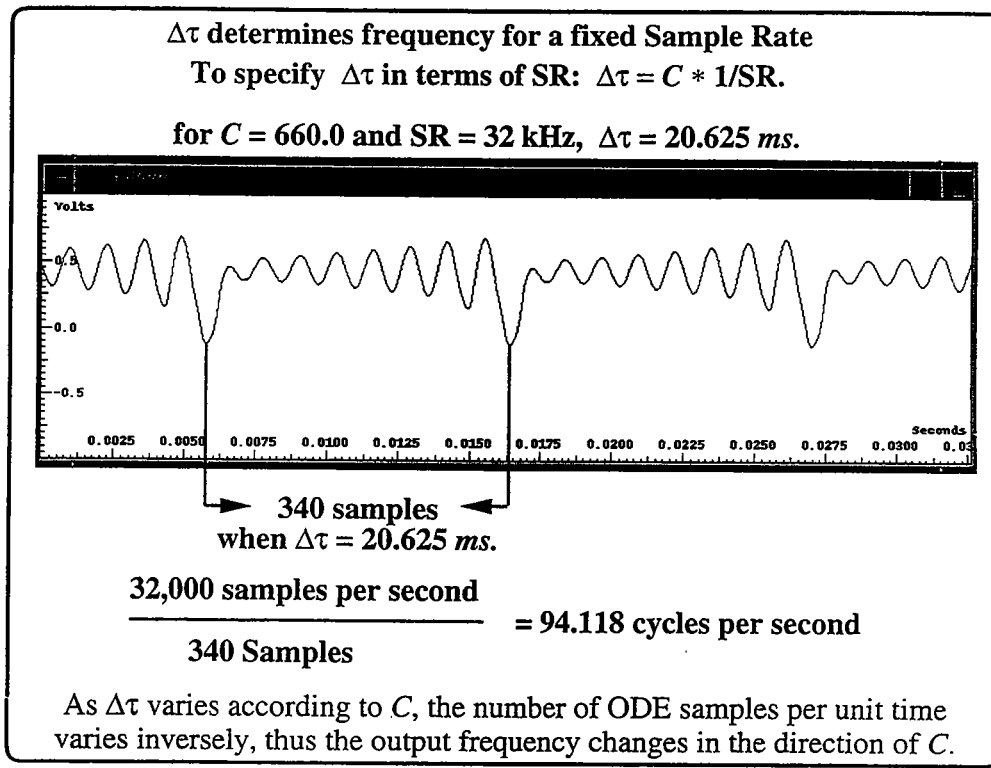


Figure 6: Waveform sample of a period-8 limit cycle from a digital simulation of Chua's circuit.  $\Delta t$  determines frequency for a given signal at a fixed sample rate. One period of the waveform computed for  $\Delta t = 20.625 \text{ ms}$  contains 340 samples, producing a 94.118 Hz frequency at a sample rate of 32 kHz.  $C$  is used to adjust  $\Delta t$  with respect to SR in order to vary the frequency of the output signal rendered at that rate.

at given instances, while rhythm accounts for the patterns of event distribution generated by listening memory.

*Conductor* sends a control signal choreographing timescaling by gesture. Conductor accounts for overall inflections in larger units such as phrases and periods as well as tempo and rhythm so the conductor's articulation extends to the formal organization of music. This function of conductor is applied to sending control signals to the computational model of the Chua's circuit through a graphic interface, the *manifold interface*. Conductor in this case is also an observer and explorer of  $n$ -dimensional control parameter space in relation to the Chua's circuit. The manifold interface records the temporal aspect as well as  $n$  number of parameter values of the control paths as drawn by the conductor's gesture.

#### 4.1 Tempo: Varying $T_i$ Applied to the Logistic Map

A rendering pipeline is designed for varying  $T_i$  while observing the logistic map. The pipeline uses the following specifications: a one to many ratio is selected, providing 6,400 samples for each iterate of the logistic map. At a sample rate of 32 kHz this creates signals of 0.2 seconds duration.  $T_i$  is initially set at 0.5 seconds (16,000 samples at 32 kHz). The synthesis engine employs frequency modulation where one sine tone, the carrier, is distorted by a second sine tone, the modulator tuned to a carrier: modulator frequency ratio of 1: 1.454545. The degree of distortion, or modulation index, is 25.0, roughly half the number of spectral energy peaks, or "partials" in the tone. For a tutorial on frequency modulation synthesis technique see Dodge and Jerse [15] and Scaletti [27]. An amplitude factor decreasing from 1.0 to 0.0 is superimposed upon each 6,400-sample duration. The resulting sound resembles a wood block. Data from each iteration of the logistic map is used to transpose the frequencies exponentially within a designated

frequency range to create a linear pitch change between iterations. Figure 7 shows the software implementation of this pipeline using the NCSA Sound Server [4]. *GTFs* are parameter-group transfer functions (*Actors* in Das [45]), *vss* houses synthesis engines [4], and *htm* provides sample scheduling for D/A conversion in real-time [44].

Varying  $T_i$  has a significant effect on sounds. Together with choice of rendering ratio,  $T_i$  determines whether the features of data will be emphasized or subdued. In this configuration,  $T_i$  sets the tempo, the perceived onset rate of discrete events. Tempo can emphasize and subdue the perception of attractors. Auditory structure of data is an outcome of a complex interaction between attractors in signals, the design of rendering characteristics enumerated above and a listener. Adjusting  $T_i$  of the period 5 logistic attractor, four sound examples were produced at three different  $T_i$  and four different stages of perceptual effects were observed.

### Experimental Reports on $T_i$

1. Perception of discrete note-events:  $T_i = 0.42$  seconds, synthesis parameter pre-set. In sound example 1, discrete note events were perceived to identify period 5 from the logistic map, meaning listeners were able to count five distinctive note events given two or three repetition of the sequence of period 5 (see Figure 8(a)).
2. Auditory grouping:  $T_i = 0.21$  seconds, synthesis parameter pre-set. In sound example 2, ears begin grouping five discrete note events into subgroups of 2 + 3, 2 + 1 + 2, producing a simple asymmetric meter. The grouping is attributed by intervallic adjacency in phase space (see Figure 8(b)).
3. Auditory streaming:  $T_i = 0.1$  second, synthesis parameter pre-set. In sound example 3, the classical auditory streaming began to emerge [14, 15]. Each repetition of the group of five note events were perceived creating a macro-unit beat, meaning the repetition of period 5 is distinctly marked and each entrance of the repetition becomes a beat. Adjacency and regular pulse were not perceptible, and three frequency regions were perceived in approximation to the lowest, uppermost and central frequency regions of the attractor in phase space (see Figure 8(c)).
4. Auditory crossfade:  $T_i = 0.1$  second, increase of amplitude onset rate and duration. In sound example 4, the perception of continuous signal flows are experienced. Discrete note events, pulse, meter, rhythm, beat, or macro-unit beat are not perceived. The woodblock timbre of the FM tone is preserved. The auditory flow is perceived as a chord-like sustained organ tone (see Figure 8(d)).

The tutorial nature of these examples is based on the preservation of the same period-5 attractor for each case. Despite their diverse acoustic character, each of these examples is generated solely by varying timescaling. No new mappings or interpolations are introduced from one example to the next. The logistic map and the timescale provide a mutual contribution to the perception of continuity. Multiple timescales help us to understand which effects may be attributed to the logistic map. Important intervallic properties between nonadjacent iterates may be observed in the overlapping spectra. There is no "correct" sound of the logistic map; instead there are informative juxtapositions of a sound production procedure and our perceptual response.

## Logistic map *tempo* rendering pipeline

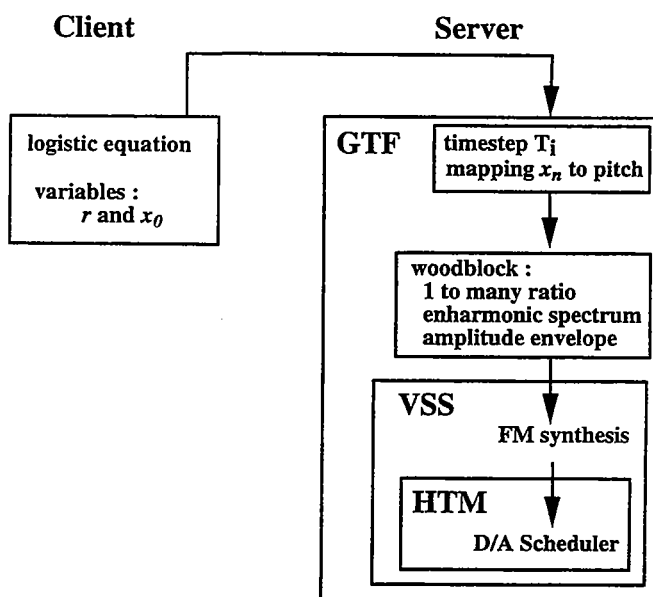


Figure 7: The software architecture of the NCSA Sound Server application, and an accompanying client application iterating the logistic equation. The client sends control messages to vary the parameter values of a frequency modulation synthesis engine. HTM is an audio sample buffer and D/A scheduler; VSS is a collection of synthesis engines; GTF's are high-level functions for generating complex sound events.

### Further Suggestion

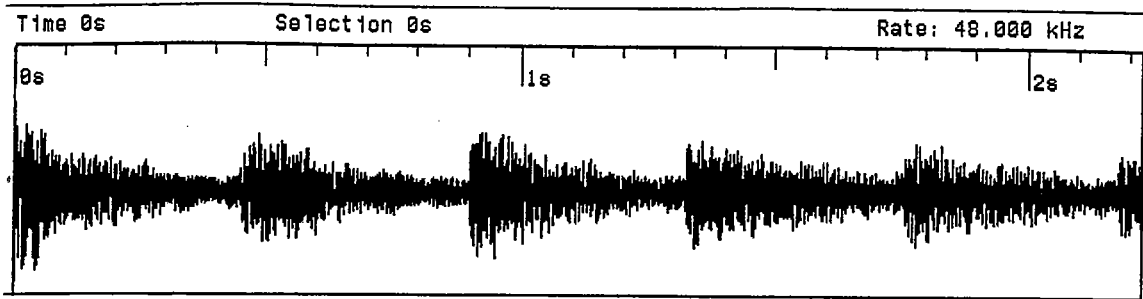
These experimental observations open the door to two directions of research. (1) We can use these sound examples at different  $T_i$  and study our perceptual responses. (2) Based upon the perceptual responses the *second order parameter* can be proposed for rescaling synthesis parameters to compensate our perceptual effects. The best known of this kind of rescaling parameter is the logarithmic scaling between frequency and pitch used in the computer music community.<sup>4</sup> Whereas the logarithmic rescaling factor is applied to one to one numerical correspondence between frequency and pitch, the second order parameter for perceiving period 5 requires as many nonlinear scales as there are synthesis parameters involved. Further the second order parameter has to be applicable at any given  $T_i$ , so that at different  $T_i$ , each iterate is perceived at equal presentation to our ears.

### 4.2 Rhythm: Embedded Observer Distributing Iterates over Time

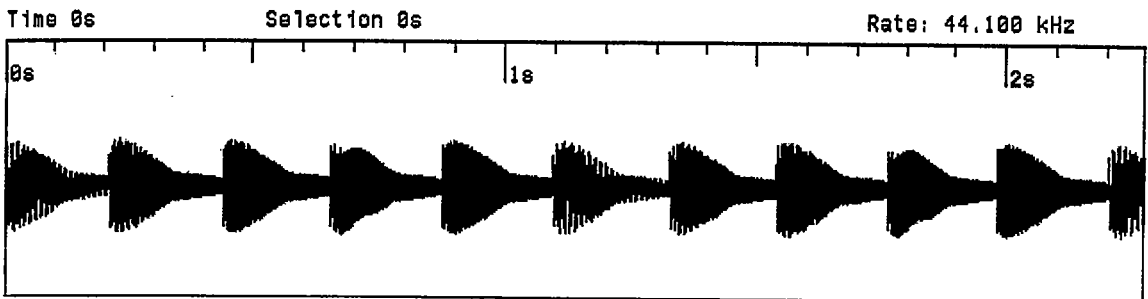
The procedure in 4.1. generates pitch sequences at a steady pulse given  $T_i$  from the logistic map. The results are music-like yet are lacking the relevance found in music between local structure and

<sup>4</sup>The perceptual response to frequency is called *pitch*. The numeric relationship between pitch and frequency is a logarithmic relationship. The pitch interval "semitone" is the frequency ratio of 1:1.05946, the twelfth-root of two, used to divide the octave, a frequency ratio of 2:1, into twelve equal parts.

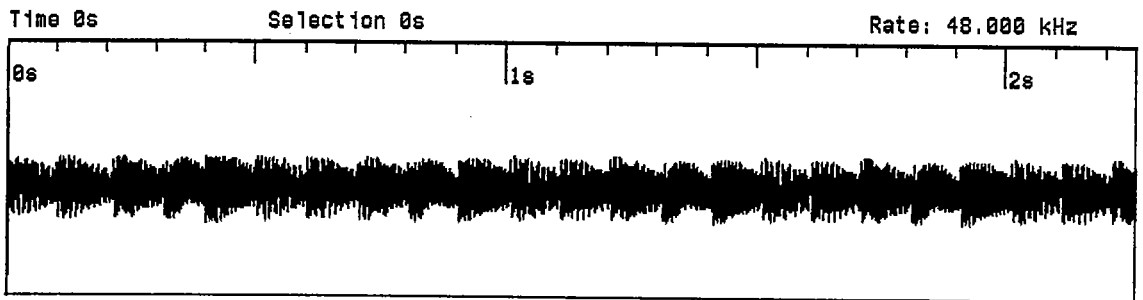
**a.**



**b.**



**c.**



**d.**

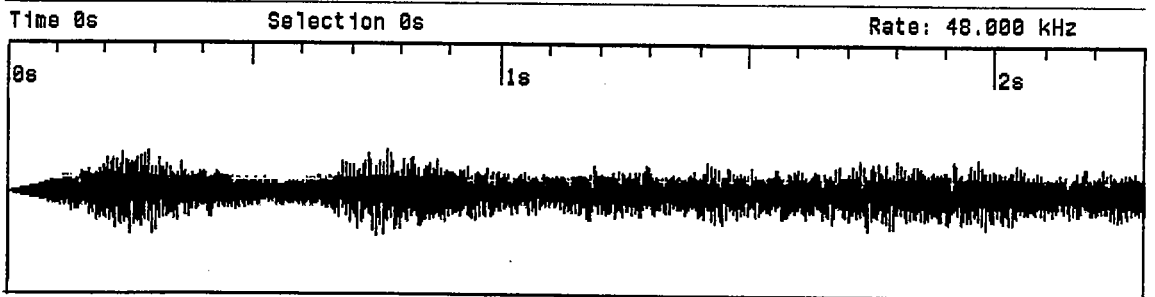


Figure 8: Time-domain display of sounds synthesized to display period-5 signals from the logistic map at different timescales.

larger structures. This method provides a local view of attractors; it is not necessarily providing a useful portrait enhancing our memory capacity for local events. Displaying every iterate at a constant duration requires an observer's unusually accurate memory to extract relevant auditory structures beyond local minima or local maxima. Also this method is somewhat acoustically impoverished: the valuable perceptual and musical attribute, rhythm is not utilized, which is another way to say silence and rest are not engaged in generating temporal grammar. As an alternative a *listening memory* has been proposed using statistical evaluation to determine relevant moments of change in an iteration sequence [28]. In this approach every iteration advances the time of the sequence by timestep  $T_i$ , each iterate is recorded and measured, and sounds are only rendered at those timesteps when relevant changes are detected. At this point we need to invent a language for various factors that contribute to temporal aspects. First we return to the two notions of time: explicit time and implicit time. Explicit time is specified by units of duration therefore is a direct measurement of time. In music, tempo, pulses, beats, and measures are examples of explicit time.  $T_i$  is an explicit time parameter. Underlying operational factors contribute to patterns in explicit time and examples of the patterns may be referred to as rhythm. We will call these underlying operational factors implicit time, because these factors do not explicitly express time or duration but they imply and contribute to determining patterns to be imprinted in explicit time. Implicit time is specified by nondurational measurement and attributed to a number of parameters applied to a particular synthesis algorithm, in this case the statistical measures.

Using *symbolic dynamics* we create sound sequences that obtain both rhythmic and timbre characteristics from the logistic map (see Figure 9).<sup>5</sup> The unit interval of the logistic map is subdivided into  $n$  bins; the number of visits to each bin are accumulated and described at each timestep as a finite sequence of  $n$  symbols, called  $n$ -cylinders. The evolving distribution of visits to various bins provides a temporal portrait of the distribution of the attractor across the unit interval. To create sounds each bin represents a set of spectral (frequency) components contributed to a tone. The distribution of visits to each bin determines the amplitude of each set of components in the output sound. As numbers of visits increase the corresponding amplitudes increase.

We have composed a method to apply rhythmic information as well as pitch and timbre to portray a chaotic system. The core idea in this approach is allowing silent iterations while a mechanism observes the system evolve, making a sound only when the observation reports a "significant change." Significance is measured using a standard statistical procedure. The mechanism involves selecting a threshold for comparing the system at each iteration. When the comparison detects a statistically significant change in distribution, all bins are sounded simultaneously. We refer to the combined spectra from all bins as a *timbre chord*. In constructing a timbre chord no information is discarded: the spectrum of the timbre chord reflects a contribution from every iterate. Attractors with sparse distributions create simple tones in comparison to sounds where every bin received a number of visits, creating a complex tone. The distribution of iterates is irregular, so rhythms are created rather than a steady pulse. The onset patterns portray the evolving system dynamics with respect to a statistical threshold. The regularity or irregularity of these rhythms, as well as their tempo indicates large-scale characteristics of attractors and tran-

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<sup>5</sup>Symbolic dynamics are a standard iterative map representation discussed for example in Peitgen et al. [29]. The norm of the difference, a standard statistical measure was proposed for a speculative discussion presented by R. Bargar at ICAD 92 and later in Mayer-Kress et al. [30]. At that time the synthesis method presented here was not fully developed, and was first utilized for the music composition *Shadowing Lemma* [31]. Sound examples from the composition were later contributed to [30] for publication. Acoustic experimental examples discussed in the present chapter were produced in September 1994 for presentation at ICAD 94. All synthesis algorithms are written in C and coauthored by the author of this chapter and Bargar (©1993).



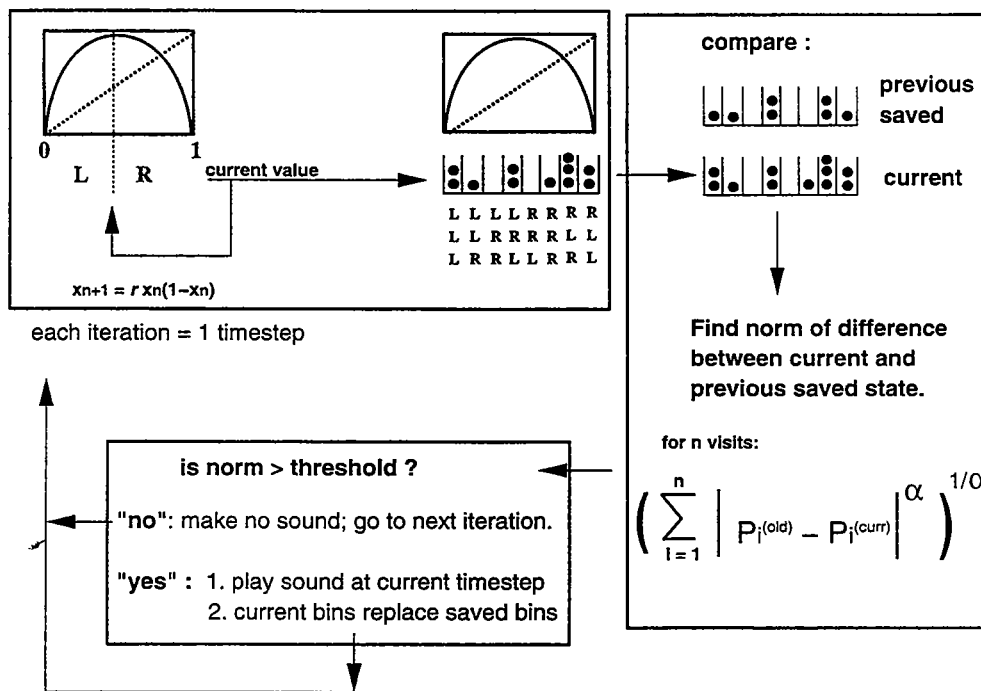


Figure 9: Algorithm for applying symbolic dynamics to generate complex timbres and rhythmic patterns from the logistic equation.

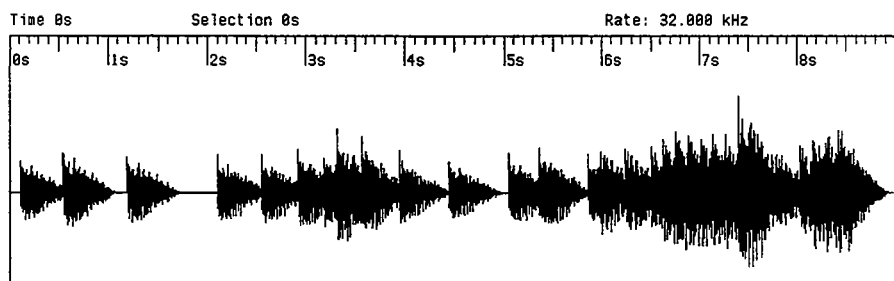


Figure 10: Onset pattern of a timeseries of timbre chords generated using the symbolic dynamics algorithm depicted in Figure 9.

sitions between attractors. Figure 10 shows the onset pattern of an evolving sequence of timbre chords.

Figure 11 shows the software implementation. This rendering pipeline provides three locations where specifications are applied: the computational model, the statistical evaluation, and the synthesis engine. In the computational model, initial  $x$  and the  $r$  parameter define the behavior of the system. For the present synthesis experiments these values were selected and remained as constants. In the statistical evaluation four parameters may be adjusted: the number of bins, the threshold for detecting change, an  $\alpha$  parameter which determines the sensitivity to different distributions, and a time-memory window to determine how many past iterations are included in each evaluation. For the present synthesis experiments the  $\alpha$  and threshold were constants, the number of bins and time-memory window were experimental variables. In the synthesis engine base frequency and frequency range were selected as variables.

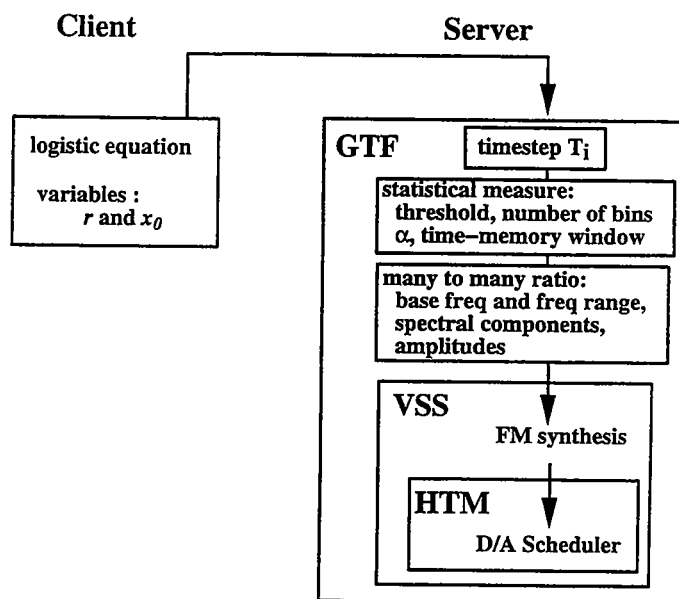


Figure 11: The NCSA Sound Server rendering timbre chords, controlled by a software implementation of the symbolic dynamics algorithm depicted in Figure 9.

## Experimental Reports on Rhythm and Timbre Chord

1. *Frequency range* of the timbre chord influences the perception of *harmonicity*. Given a fixed number of bins, the variation of the frequency range varies the degree of the distinctiveness of each bin. When one wishes to monitor the registration activities of iterates of one bin to another, frequency range has to be assigned adequately in consideration of the number of bins. 16 bins of information distributed along the range of one octave will be perceived as less distinctive to the same information distributed along the frequency range of 3 or more octaves. Accordingly monitoring 4 bins of information would not require the frequency range of 5 octaves unless it is preferred. A frequency range less than one-half octave subdues distinct emergence of individual of bins for more than four bins.
2. *Base frequency* of the timbre chord transposes the frequency range. Given 200 Hz as base frequency and 4 octaves as frequency range, the frequency spectra of timbre chords will range from 200 Hz to 3200 Hz which falls within the good range for our ears for frequency differentiation—our ears fairly process the frequency information ranging from 20 Hz to 4000 Hz [32]. The selection of base frequency has to be made in consideration of frequency range and the ear's physiological capability. As the base frequency goes below 200 Hz the pitch and spectral characteristics will be less emphasized especially for a narrow frequency range, approaching percussive emphasis of rhythm. As the base frequency increases above 1600 Hz pitch and timbre are less emphasized for any frequency range.
3. *Number of bins* affects both timbre and rhythm. Bins increase and decrease in units of  $2^n$ . Complex tone clusters with small changes in internal detail are characteristic of  $n > 3$  when  $s < 2^n$ , where  $s$  is the number of semitones within the frequency range. For  $s > 2^n$  individual bins are more distinct. As  $n$  increases regardless of  $s$  the threshold for detecting significant change becomes oriented to changes distributed evenly across the histogram rather than to catastrophic changes in a few regions of the histogram.
4. *Statistical window size* determines a large-scale or local portrait of dynamical behavior. As

the attractor stabilizes in a convergent region it tends to undergo a decrease in statistical significant change. Reducing change results in a characteristic decrease in sound events. A time-memory window for discarding old iterates from statistical evaluation will allow the statistics to be refreshed and focus attention on local change. The characteristic decrease in sound events is avoided or delayed.

### Further Suggestion

The selection of statistical measure provides the dominant influence over the rhythmic character of the sequence. This method provides a portrait of the measuring tool as well as the portrait of dynamics of particular logistic values. We can select two modes of study: (1) select invariant statistical measure parameters and vary  $r$  or  $x_0$  observing characteristics of different attractors; (2) select a logistic attractor and keep it constant while varying the statistical measure, observing different *listening memory* characteristics. It suggests this method may be applied for the study and evaluation of influence of statistical methods upon observed dynamics in many systems.

### 4.3 Conductor: Gesture-Based Interactive Timescaling for Sending Control Signals

For immediate feedback from timescale exploration of a chaotic system, explicit and implicit time parameters may be varied in real-time by an observer. An observer controlling the changes in the states of a system in real-time can generate a temporal observation. Gestures create auditory events, and we refer to the velocity of a gesture as *event time*  $T_e$ , which expresses the rate a parameter value is varied under control of a gesture. We use  $\Delta e$  to refer to the change of  $T_e$  during an event, accounting for the speed variation of an observer's control, which is seldom constant in a gesture.  $T_e$  and  $\Delta e$  are guided by auditory feedback as an observer listens for features and explores details. An understanding of the system dynamics comes from comparing the control rate  $T_e$  with the rate of change of the auditory signal. The observer develops an intuitive sense of  $T_e$  since she or he is generating gestures and receiving immediate auditory feedback responding to her or his actions. The observer can measure the robustness of the state changes caused by  $T_e$  by introducing fine variations in the control rate ( $\Delta e$ ). Gestures may also be recorded and numerically measured in comparison to changes in the auditory signal.

The technique of parameter variation is among the simplest methods to influence chaotic signals. By changing the resistance, capacitance, and inductance of the circuit components the state of the system will be changed according to its internal principles. Since the first confirmation of chaos in an experimental circuit it has often been observed that changing the values of circuit components can induce sequences of bifurcations and chaotic signals [34, 35]. These bifurcations can produce significant pitch, loudness, and timbre variation. We have explored methods for generating specific sequences of auditory signals from the Chua's circuit by applying transient control signals to multiple parameters, using time-specific, interactive methods to capture and reproduce users' control gestures.

#### 4.3.1 Controlling the Chua's Circuit

To study effects of parameter variation a circuit is needed with components capable of receiving control voltages from an external source. Using the simulation of the Chua's circuit, parameter variation was implemented as a test case and the knowledge obtained was later applied to the design and implementation of a voltage controlled analog circuit. For the detailed description of the implementation of the control of the analog circuit we refer to the paper by Zhong, Bargar, and

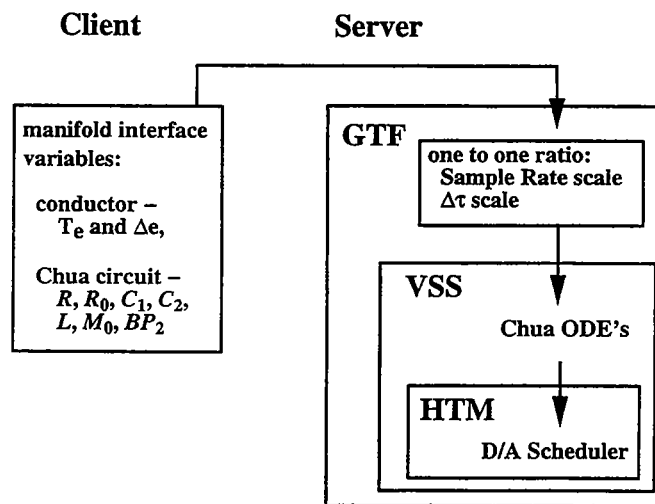


Figure 12: The NCSA Sound Server simulation of the Chua's circuit, and an accompanying interactive interface.

Halle [25].<sup>6</sup> Figure 12 shows the software implementation of the Chua's circuit simulation in a real-time interactive sound synthesis architecture. To make these observations the six parameters  $C_1, C_2, L, R, R_0, BP_2$  and the  $G_a - G_b$  slope are variable in real-time, providing immediate auditory feedback. To help articulate the control space of the Chua's circuit, upper and lower parameter boundaries may be specified interactively using the mouse and keyboard. This provides scaling from a control space to the range of each component being controlled. Auditory feedback is often consulted for selecting these control boundaries. Auditory structures may be described in terms of *spectral focus* and *timbre rhythm*.

## Experimental Reports of Conductor-Centered Observations

1. A *control path* is an event defined by a trajectory in control space and in time. Trajectories are defined by the gestures an observer makes using an external hardware interface such as *mouse*. In virtual reality implementation an event is controlled with 3-degrees of freedom and its trajectory is defined in 3-D according to the observer's physical movements with a hardware interface called a *wand*. In desk-top computer implementation, an event is controlled with 2-degrees of freedom and its trajectory is defined in a three coordinate system with one coordinate drawn as a function of the other two coordinates; the control path is drawn using a *mouse* and movement of the mouse is constrained in 2-D. Events are generated for making comparisons between multiple states in a simulation, or comparisons between parameter changes and state changes. Control paths are created during real-time observations and recorded in control space and in time.

A path encodes an observation itself as an event to the extent that the event time defined by  $T_e$  is recorded in the path and one can observe, when retrieving the path,  $T_e$  varies according to how fast or slow the observer was drawing the control path. This variation of  $T_e$  implies two things; (1) in terms of an observer's state of observation—when it slows down around

<sup>6</sup>The implementation of the voltage controllable analog circuit was designed and built by G. Q. Zhong at UC Berkeley, and this device became an important instrument for acoustic exploration and realization of the composition, *anti-Odysseus* [33]. Voltage-scaling components and a metal housing box with switches were provided by CCSR at the University of Illinois. For original observation of chaos in an experimental circuit study by Zhong and Ayrom [36, 37].

the region of periodic attractor it implies the observer was searching for a stable boundary via her or his auditory feedback, (2) in terms of acoustic results—as it moves slowly the associated bifurcation sequence occurs accordingly reflected in acoustic results. We hear series of sounds changing the complexity and timbre as the state changes. As the observer's gesture sweeps through the control path at fast speed the chaotic system does not have sufficient time to settle into any associated stable states; in the series of changes we will hear perceptually continuous sounds with unsettled transient quality.

The states that can be achieved depend on part upon the resolution of the  $\Delta$  applied to parameters being varied. In the graphical interfaces  $\Delta$  is defined in terms of a parameter range. For a given length of a control path, a small parameter range defines a fine  $\Delta$ , a large parameter range defines a coarse  $\Delta$ . The definition of an event by a gesture we feel is an important temporal unit for making observations. Listening while controlling  $T_e$  tends to encourage searching for states according to their associated acoustic results. We include  $\Delta_e$  as a relevant variable because searching often employs  $\Delta_e$  during an event to define regions of greater and lesser interest in a control path. In effect  $\Delta_e$  introduces a temporal grammar to subdivide a gesture, to define meaningful paths within a continuous control space.

2. *Spectral focus* is a characteristic of auditory structures related to the complexity of an auditory signal. Spectral focus encompasses the frequency-domain characteristics *harmonicity* and *spread*. Human ears are sensitive to energy distribution in the spectrum of signals such as can be seen in an STFT of a signal. Energy peaks indicate auditory structures and can be defined by their amplitude and their frequency distribution in the spectrum. *Harmonicity* describes energy peaks in relation to one another: the ratio between frequencies of energy peaks describes the complexity of the signal. As a general case, simpler ratios make simpler signals. *Spread* describes the frequencies of energy peaks in relation to the frequency range of the signal. A wide spread of energy can indicate a bright timbre or the presence of multiple pitch regions, a characteristic of bifurcations. *Searching* a control space involves acoustic orientation to the complexity and character of the signal. We have observed control paths slowing down as listeners focus on the details of regions according to acoustic criteria they have chosen. The criteria do not require external specification, they are arrived at by listening, encoded within a control path, and articulated by  $\Delta_e$ . By re-examining recorded control paths and their effects on a simulation we become observers to another observer's interactions with the simulation. And our experience of time measure applied to the simulation involves both the signals from the simulation and the temporal grammar of the control path event.
3. *Timbre rhythm* is produced by an interaction between the perceived continuity of an auditory signal and discontinuities within the signal. Bifurcations are responsible for timbre rhythm, generating discontinuities that are rendered into a continuous auditory signal resulting in ambiguous temporal characteristics. This ambiguity produces a sound having a rhythmic presentation which is also a timbral presentation. Irregularity of discontinuities creates rhythms; when rhythms occur in sufficiently rapid succession they create timbre. The timbre is fine details of the sounds in which the rhythmic characteristics are migrated into the microstructure of the sounds. When there is an ambiguity due to temporal unit wherein the unit is not long enough to classify its acoustic results in terms of rhythm and not short enough to attribute to the timbre we achieve sounds with timbre-preserving rhythmic characteristics. Unlike steady-state or periodic waveforms, these timbres include a rhythmic roughness. The chaotic property known as intermittency also generates timbre rhythm. Intermittency

occurs when two or more states alternate. Sparse intermittency involves a primary spectrum interrupted by bursts of alternative spectra, creating rhythmic patterns. As intermittency becomes less sparse the bursts merge and create a timbre. The signal no longer sounds like a rhythmic pattern, however imbedded in the timbre are rhythmic characteristics.

## Further Suggestion

Unique bifurcation scenarios may be generated using multiple-parameter variation. Figure 13 shows a *period-adding bifurcation sequence* generated in the Chua's circuit by linearly increasing a single parameter,  $BP_2$ . Between each period is located an intermittency region. Period-doubling creates a descending tone-sequence as the periods increase in length. Effects of controlling individual parameters suggested further research [33, 36] which indicates the ability to navigate multiple parameters of a simulation greatly enhances our ability to observe simulation dynamics. At the same time it is auditory feedback that provides the guide for navigation identifying "landmarks" of the state of the system. This suggests an increased ability to interface with multiple parameters and control their real-time variation will increase our ability to explore the states of a simulation and to articulate structures implicit in those states. In section 5 we discuss an example of an interactive interface which implements control paths and control spaces as visual three-dimensional objects.

## 5 Capturing Gestures by Intuitive Graphical Interface

Recently an interface has been developed for simultaneous control of multiple parameters of real-time software simulations [36]. We demonstrate this interface using the simulated Chua's circuit. A graphical interface helps to physically manipulate the simulation and to visually identify parameter regions that produce acoustically interesting signals. The virtual environment implementation of this interface to enhance navigation in immersive environments was presented in Bargar et al. [40].

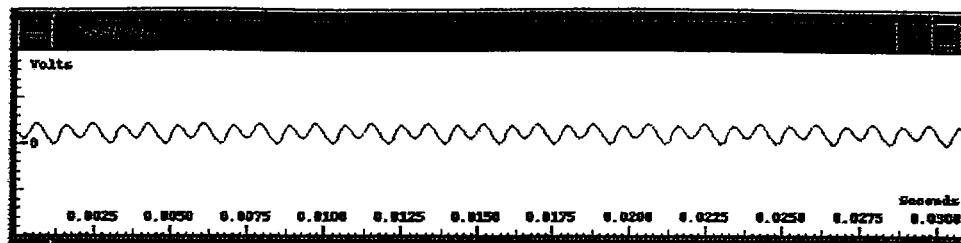
The manifold interface was developed to meet the need for an intuitive navigation of the multidimensional control space (see Figure 14). An arbitrary number of  $n$  circuit parameters can be controlled from a graphical surface by defining arbitrary functions for converting manifold position values into circuit control voltage values. The surface indicates positions in  $n$  manifold dimensions using three graphical coordinates. A manifold surface is not specific to the Chua's circuit simulation, it is an interface and composition tool embedded in a cube which defines projections of control values for  $n$  parameters; in the Chua's circuit case these are the six physical parameters  $\{R, R_0, C_1, C_2, BP_+$  (also called  $BP_2$ ) and  $m_0$  (also called  $G_a$ ) $\}$  [25]. Each axis of the cube provides a linear control dimension. One or more parameters may be controlled from each cube dimension. For each parameter the corresponding axis endpoints of the cube are assigned an upper and lower parameter value. In this way each cube defines a set of parameter range relations and ratios for parameter changes. Traversing the parameter space of the cube results in changes to the associated circuit parameters according to the functional definition of the cube axes.

To help specify regions in the cube where desired parameter combination occur, control signals from the cube are obtained from surfaces within the cube. A surface is defined by a continuous edge, a set of control points, and spline functions<sup>7</sup> between control points. A single surface does

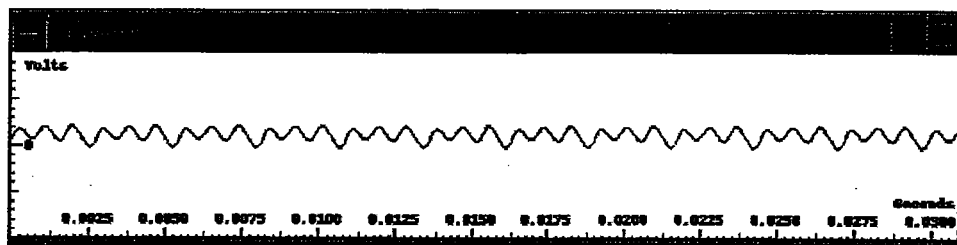
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<sup>7</sup>Spline functions are a class of curves that are generated in relation to  $n$  control points. Spline curves originated from a graphic design technique for drawing smooth curves, by bending a flexible metal rule to fit with pressure against a number of pins that have been pushed into the drafting table. Some splines are affected only locally by control points; other splines are influenced by every control point at every point along the curve. Depending upon

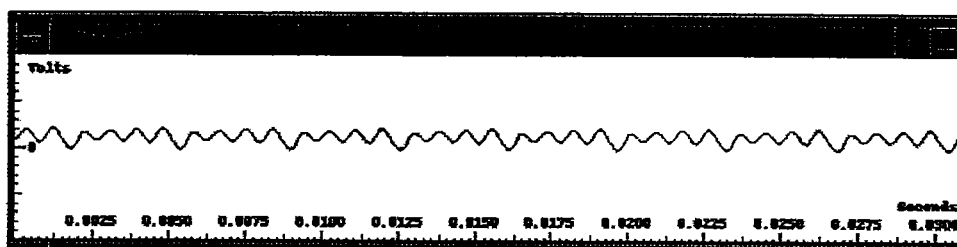
### period-two



### period-three



### period-four



### period-six

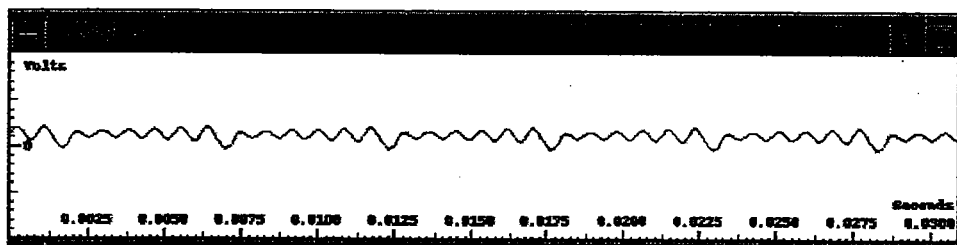


Figure 13: Waveform samples from a period-adding bifurcation sequence from Chua's circuit. Not shown here is period-five, which also occurs in the sequence. The periodic limit cycles become increasingly unstable as the length of the period increases.

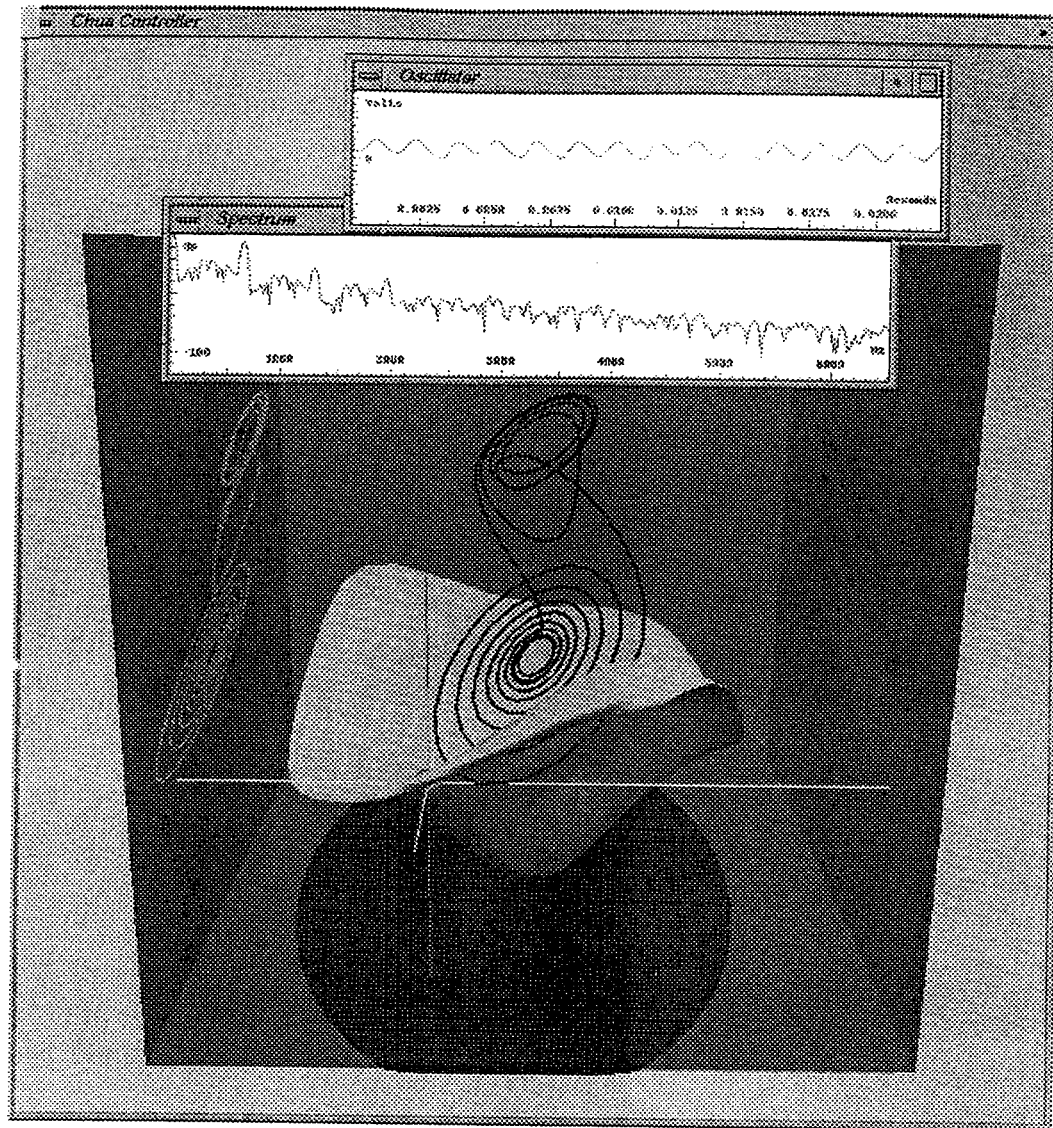


Figure 14: A *manifold interface* designed to control an arbitrary high-dimensional parameter space, applied to control the Chua's circuit. The graphical surface selects a subset of the control space represented by the cube. The spherical cursor located at the crosshairs may be maneuvered in real-time to select specific parameter values for the circuit. The path of the cursor movements on the surface may be recorded and those interactive gestures reproduced automatically.



not provide access to all possible parameter combinations in the cube, only to a subset selected by the composer. Decisions are made to position a surface in a cube by auditioning the sound signals generated at selected surface points. More than one surface may be defined within a cube.

Surfaces provide visual cues for potential trajectories in control space. Visual characteristics of a surface can become associated in a user's experience with specific sounds. Moving on a surface, the software allows a user to draw control paths while listening to their acoustic result. Here drawing a path is equivalent to defining a trajectory in an n-dimensional control space. Paths record both the trajectory position and the speed of the drawing gesture. A user can "replay" a path and make adjustments to its rate and position. Recorded paths may be stored in computer memory and recalled. Paths may be combined to construct larger acoustic sequences. In the Chua's circuit case the rate of path traversal effects the resulting quality of the acoustical signal; the harmonic stability of a waveform can vary depending upon the speed with which a circuit signal converges onto an attractor or transitions from one attractor to another, and upon the speed with which new circuit states are introduced by parameter variation. Configuring Chua's circuit and the manifold interface in an interactive musical performance system is described in detail in Choi [38].

## 6 Concluding Remarks

For bringing the behavior of computational models into auditory signals, timescaling variables are attributed either to a computational model or to a synthesis engine. The notions of explicit and implicit time have been introduced in terms of the corresponding functions of timescaling variables. A class of sound examples is generated from each of three different rendering pipelines provided for computational models of the logistic map, the logistic map with listening memory, and the Chua's circuit (see Table 2). To address the rendering problems posed by these models, the timescaling principles tempo, rhythm and conductor are introduced. Experimental reports made from these sound examples provide working vocabularies appropriate to discuss the variety of auditory structures emerging from different timescaling principles. Whereas tempo is attributed as an explicit time variable, rhythm is a resultant pattern in explicit time from the functions of implicit time variables. The function of a conductor in timescaling is gesture-based interactivity with a computational model for sending control signals, with immediate auditory feedback for modifying control signals. The conductor creates an event time based upon the immediate auditory feedback. As presented in section 4, the application of these timescaling principles are related to the rendering ratios between computational models and synthesis engines which are described in terms of one to many, many to many, and one to one.

The temporal grammar involving silences as well as sounds bridges computational models and synthesis engines and can be varied from one application to another. One may speculate whether we can generate a universal principle for the temporal grammar for sonification projects. The author can report that there is such a thing approaching a "universal grammar" in well defined western music practice during the period roughly from 1750 to 1900, often referred to as the common practice period. Whether this speculation of our imaginary speculator is relevant to sonification projects or not relevant, it is certainly innocent and possibly premature at this stage since the projects have not received enough practice. Sonification projects can be practiced only by producing sounds and listening to the sounds. The suggestion is that becoming a student of sounds and learning to be an attentive listener is the first step to sonification. This suggestion is

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the particular spline, the curve may or may not be required to pass through each of the control points.

computational model	function type	time characteristic of comp. model	applied timescale attributes	rendering ratio	applied synthesis timescaling	synthesis engine	auditory signal
logistic map	iterative	implicit time	$T_i$ (explicit)	one to many	enharmonic spectra; woodblock ampl. env.	frequency modulation	discrete
logistic map	iterative	implicit time	$T_i$ (explicit); statistical measure (implicit)	many to many	timbre chord spectra; amplitude envelopes	frequency modulation	discrete
Chua circuit	continuous	explicit time	$\Delta\tau = C * (1/SR)$ (explicit)	one to one	Chua signal to SRate; conducted control signal events	Chua ODE's	continuous*

Table 2: \*includes silences at fixed points and discontinuities at bifurcations.

presented as an alternative to imaginary sounds and systems which are elaborated in the absence of experimental examples or sound synthesis.

Based upon the complexity of computational models, different classes of problems have to be addressed in designing synthesis engines. Making distinctions between explicit and implicit time variables when attributing timescaling factors to computational models, and specifications for rendering ratios when associating data from computational models to synthesis parameters, are the key factors for generating sounds discussed in this chapter. Future projects foresee generating more cases through sounds to help to specify classes of second-order parameters for efficient timescaling application.

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